ONLINE APPENDIX TO:

"Do Matching Frictions Explain Unemployment? Not in Bad Times."

Pascal Michaillat[*]

A1 Proofs

Proof of Propositions 1, 2, and 3. Equilibrium condition (10) uniquely defines θ as an implicit function of c in a static environment in the canonical model. Assume that there exists $L \geq 0$ such that $\theta < L$ for all $c > 0$. As $1/q(\cdot)$ increases in θ,

$$0 < \left\{ [1 - \delta \cdot (1-s)] \frac{1}{q(\theta)} + \delta \cdot (1-s) \cdot \beta \cdot \theta \right\} < \left\{ [1 - \delta \cdot (1-s)] \frac{1}{q(L)} + \delta \cdot (1-s) \cdot \beta \cdot L \right\} \equiv \lambda.$$

Condition (10) cannot hold for

$$0 < c < \frac{1}{\lambda} \cdot (1 - \beta).$$

Thus $\lim_{c \to 0} \theta(c) = +\infty$ and $\lim_{c \to 0} f(\theta(c)) = 1$. Equation (3) implies $\lim_{c \to 0} n(c) = 1$. I use a similar argument to prove Proposition 3 but I exploit equilibrium condition (13) instead of condition (10). Finally, I use the same argument to prove Proposition 2 but I exploit equilibrium condition (12) instead of condition (10). (I also use the fact that $n \leq 1$ such that $n^{1-\alpha} \leq 1$). □

Proof of Proposition 4. When $a \geq a^R$, I follow the proof of Proposition 2 but I exploit equilibrium condition (14) instead of condition (12). When $a < a^R$, the result follows from the continuity of all functions involved in equilibrium condition (14). □

Proof of Proposition 5. Assume that $a \in (0, a^R)$. Let $n^F \equiv n^R - n$. I rewrite (14) as

$$\alpha \cdot \left\{ n^{\alpha-1} - \left(n^R \right)^{\alpha-1} \right\} = [1 - (1-s) \cdot \delta] \cdot \frac{c}{q(\theta)}$$

$$\alpha \cdot (1-\alpha) \int_0^{n^F} \left(n^R - x \right)^{\alpha-2} dx = [1 - (1-s) \cdot \delta] \cdot \frac{c}{q(\theta)}.$$

[*]Department of Economics, London School of Economics, Houghton Street, London, WC2A 2AE, UK. Email: p.michaillat@lse.ac.uk. Web: https://sites.google.com/site/pmichaillat/.

Differentiating the last line with respect to a yields

$$\left\{\alpha\cdot(1-\alpha)\cdot(2-\alpha)\int_0^{n^F}\frac{\partial n^R}{\partial a}\cdot\left(n^R-x\right)^{\alpha-3}dx\right\}+\left\{\alpha\cdot(1-\alpha)\cdot\frac{\partial n^F}{\partial a}\cdot n^{\alpha-2}\right\}$$

$$=-c\cdot[1-(1-s)\cdot\delta]\cdot\frac{\partial q}{\partial\theta}\cdot\frac{1}{q(\theta)^2}\cdot\frac{\partial\theta}{\partial a}.$$

Equations (14) and (3) yield $\partial n/\partial a>0$ and $\partial\theta/\partial a>0$. Furthermore, $\partial q/\partial\theta<0$ and $\partial n^R/\partial a<0$ when $a\in(0,a^R)$. Using the equation above infer that $\partial n^F/\partial a>0$. I conclude using these comparative statics as well as the relationships $u(a)=1-(1-s)\cdot n(a)$, $u^F(a)=s\cdot n(a)+n^F(a)$, and $u^R(a)=1-n^R(a)$. $\qquad\square$

Proof of Proposition 6. Let $\check{x}=d\ln(x)$. Note that $\check{x}/\check{y}=d\ln(x)/d\ln(y)=\varepsilon_y^x$. Under Assumptions 3 and 5 the firm's optimality condition is given by (14). I differentiate (14) around the static equilibrium for a small variation in recruiting cost c:

$$\frac{\alpha\cdot n^{\alpha-1}\cdot q(\theta)}{c\cdot[1-(1-s)\cdot\delta]}\cdot(\alpha-1)\cdot\check{n}=\check{c}+\eta\cdot\check{\theta}$$

$$\varepsilon_c^\theta=-\left[\eta+(1-\eta)\cdot(1-\alpha)\cdot u\cdot T(\theta,n)\right]^{-1},$$

where $T(\theta,n)\equiv\alpha\cdot q(\theta)\cdot n^{\alpha-1}/(c\cdot[1-\delta(1-s)])$. From the Beveridge curve (3) I inferred that $\varepsilon_c^n=\varepsilon_\theta^n\cdot\varepsilon_c^\theta=(1-\eta)\cdot u\cdot\varepsilon_c^\theta$. From the system characterized by the firm's optimality condition (14) and the Beveridge curve (3), $d\theta/da>0$, $dn/da>0$, and $du/da<0$. Moreover, $\partial T/\partial n<0$, $\partial T/\partial\theta<0$ so $dT/da<0$. Hence, $d|\varepsilon_c^\theta|/da>0$.

Note that if $a<a^R$, $\alpha\left(n^R\right)^{\alpha-1}=w/a$. Moreover, using the firm's optimality condition (14),

$$T(\theta,n)=\frac{\alpha\cdot n^{\alpha-1}}{[1-\delta\cdot(1-s)]\cdot c/q(\theta)}$$

$$T(\theta,n)=\frac{\alpha\cdot n^{\alpha-1}}{\alpha\cdot n^{\alpha-1}-\alpha\cdot\left(n^R\right)^{\alpha-1}}$$

$$T(\theta,n)=\frac{1}{1-\left(n^R/n\right)^{\alpha-1}}.$$

From definition (18), $u^F=s\cdot n+(n^R-n)$. If s is small enough (if the time period is short enough), $u^F\approx n^R-n$ so $n\approx n^R-u^F$. If $u<<n$, $u^F<<n^R$, and $n^R\approx1$. In that case,

$$T(\theta,n)\approx\frac{1}{1-\left(1-\frac{u^F}{n^R}\right)^{1-\alpha}}\approx\frac{1}{(1-\alpha)}\cdot\frac{n^R}{u^F}$$

$$\varepsilon_c^\theta\approx-\left[\eta+(1-\eta)\cdot u\cdot\frac{n^R}{u^F}\right]^{-1}\approx-\left[\eta+(1-\eta)\cdot\frac{u}{u^F}\right]^{-1}.$$

$\qquad\square$

Proof of Lemma 1. Let L_t denote the value to a worker of being employed after the matching process in period t. Let U_t denote the value to a worker of being unemployed.

$$L_t = w_t + \delta \cdot \mathbb{E}_t \left[\{1 - s \cdot (1 - f(\theta_{t+1}))\} L_{t+1} + s \cdot (1 - f(\theta_{t+1})) \cdot U_{t+1} \right]$$

$$U_t = \delta \cdot \mathbb{E}_t \left[(1 - f(\theta_{t+1})) \cdot U_{t+1} + f(\theta_{t+1}) \cdot L_{t+1} \right].$$

These continuation values are the sum of current payoffs, plus the discounted expected continuation values. Combining both conditions yields the worker's surplus from an established relationship with a firm:

$$L_t - U_t = w_t + \delta \cdot \mathbb{E}_t \left[(1 - s) \cdot (1 - f(\theta_{t+1})) \cdot (L_{t+1} - U_{t+1}) \right].$$

The firm's surplus from an established relationship is the hiring cost $c \cdot a_t / q(\theta_t)$ because a firm can replace a worker at that cost during the matching period. Assume that wages are continually renegotiated. Since the bargaining solution divides the surplus of the worker-firm match with the worker keeping a fraction $\beta \in (0, 1)$ of the surplus, the worker's surplus is related to the firm's surplus and

$$L_t - U_t = \frac{\beta}{1 - \beta} \cdot \frac{c \cdot a_t}{q(\theta_t)}.$$

Thus the solution of the bargaining game is

$$w_t = c \cdot \frac{\beta}{1 - \beta} \cdot \left\{ \frac{a_t}{q(\theta_t)} - \delta \cdot (1 - s) \cdot \mathbb{E}_t \left[\left(\frac{1}{q(\theta_{t+1})} - \theta_{t+1} \right) \cdot a_{t+1} \right] \right\}.$$

$\square$

Proof of Lemma 2. The wage schedule $w(n_t(i))$ is determined by Nash bargaining over the surplus from the marginal match. To simplify, I assume that the wage that solves the bargaining problem does not generate layoffs. I verify at the end of the derivation that the solution actually satisfies this condition. As in the proof of Lemma 1 the worker's surplus from being employed in firm i is

$$L_t - U_t = w(n_t(i)) + \delta \cdot \mathbb{E}_t \left[(1 - s) \cdot (1 - f(\theta_{t+1})) \cdot (L_{t+1} - U_{t+1}) \right]. \tag{A1}$$

Once the recruiting expenses are sunk, the surplus J_t accruing to the firm from hiring a marginal worker is the marginal profit of having an additional worker:

$$J_t = \frac{\partial g}{\partial n(i)} (n_t(i), a_t) - w_t(n_t(i)) - n_t(i) \cdot \frac{\partial w}{\partial n(i)} (n_t(i)) + (1 - s) \cdot \delta \cdot \mathbb{E}_t \left[\frac{c \cdot a_{t+1}}{q(\theta_{t+1})} \right]. \tag{A2}$$

The first-order condition of the firm's profit-maximization problem implies

$$J_t = \frac{c \cdot a_t}{q(\theta_t)}. \tag{A3}$$

Since the bargaining solution divides the surplus of the match between the marginal worker and the firm with

3

workers keeping a fraction $\beta \in (0,1)$ of the surplus, worker's surplus is related to firm's marginal surplus by

$$L_t - U_t = \frac{\beta}{1-\beta} \cdot J_t. \tag{A4}$$

I combine (A1)-(A4) to derive a differential equation for the wage schedule:

$$w(n_t(i)) + \beta \cdot n_t(i) \cdot \frac{\partial w}{\partial n(i)}(n_t(i)) = \beta \left[\frac{\partial g}{\partial n(i)}(n_t(i), a_t) + c \cdot (1-s) \cdot \delta \cdot \mathbb{E}_t \left[a_{t+1} \cdot \theta_{t+1} \right] \right].$$

With $g(n_t(i), a_t) = a_t \cdot n_t(i)^\alpha$ the solution of the differential equation is

$$w(n_t(i)) = \beta \cdot \left[\frac{\alpha \cdot a_t \cdot n_t(i)^{\alpha-1}}{1 - \beta \cdot (1-\alpha)} + c \cdot (1-s) \cdot \delta \cdot \mathbb{E}_t \left[a_{t+1} \cdot \theta_{t+1} \right] \right].$$

$\square$

Proof of Lemma 3. I determine a condition on the stochastic process for technology as well as the parameters of the model such that private efficiency of all worker-firm matches be respected at all time. By Definition 2, a necessary and sufficient condition for private efficiency is

$$0 \leq w_t \leq \frac{\partial g}{\partial n(i)}(n_t^*, a_t) - n_t^* \cdot \frac{\partial w}{\partial n(i)}(n_t^*, \theta_t, n_t, a_t) + \delta \cdot (1-s) \cdot \mathbb{E}_t \left[\frac{c \cdot a_{t+1}}{q(\theta_{t+1})} \right], \tag{A5}$$

where $n_t^* \equiv (1-s) \cdot n_{t-1}$. Under Assumptions 3 and 5, condition (A5) becomes

$$\omega \cdot a_t^\gamma \leq \alpha \cdot a_t \cdot [(1-s) \cdot n_{t-1}]^{\alpha-1} + \delta \cdot (1-s) \cdot \mathbb{E}_t \left[\frac{c \cdot a_{t+1}}{q(\theta_{t+1})} \right].$$

Since $\mathbb{E}_t \left[\frac{c \cdot a_{t+1}}{q(\theta_{t+1})} \right] \geq 0$, a sufficient condition for private efficiency is

$$\frac{\alpha}{\omega} \cdot (1-s)^{\alpha-1} \cdot n_{t-1}^{\alpha-1} \geq a_t^{\gamma-1}. \tag{A6}$$

The firm's optimality condition equation in period $t-1$ in a symmetric equilibrium is

$$\alpha \cdot a_{t-1} \cdot n_{t-1}^{\alpha-1} = \omega \cdot a_{t-1}^\gamma + \frac{c \cdot a_{t-1}}{q(\theta_{t-1})} - \delta \cdot (1-s) \cdot \mathbb{E}_{t-1} \left[\frac{c \cdot a_t}{q(\theta_t)} \right].$$

I assume that technology is persistent enough such that for all $t-1$

$$\frac{c \cdot a_{t-1}}{q(\theta_{t-1})} - \delta \cdot (1-s) \cdot \mathbb{E}_{t-1} \left[\frac{c \cdot a_t}{q(\theta_t)} \right] \geq 0.$$

This is equivalent to imposing that frictional unemployment always be positive (this is verified in the simu-

lations of the calibrated model). Under this technical assumption,

$$\alpha \cdot a_{t-1} \cdot n_{t-1}^{\alpha-1} \geq \omega \cdot a_{t-1}^{\gamma}. \tag{A7}$$

Plugging (A7) into (A6) yields the following sufficient condition:

$$(1-s)^{\alpha-1} \geq \left(\frac{a_{t-1}}{a_t}\right)^{1-\gamma}$$

$$(\alpha-1) \cdot \ln(1-s) \geq (1-\gamma) \cdot \left[\ln(a_{t-1}) - \ln(a_t)\right].$$

Using $\ln(a_t) = \ln(a_{t-1}) + z_t$, I find that a sufficient condition to avoid inefficient separations in period t is

$$z_t \geq \frac{1-\alpha}{1-\gamma} \cdot \ln(1-s).$$

Let $\Phi(\cdot)$ be the cumulative distribution function of the $N(0,1)$ distribution. Given that z_t is normally distributed with variance σ^2, inefficient separations occur with probability below

$$\Phi\left(\frac{1}{\sigma} \cdot \left[\frac{1-\alpha}{1-\gamma} \cdot \ln(1-s)\right]\right).$$

Conversely if we want inefficient separations to occur with a probability below p, it is sufficient that the wage flexibility γ verifies

$$\gamma \geq 1 - (1-\alpha) \cdot \frac{\ln(1-s)}{\sigma \cdot \Phi^{-1}(p)}$$

$\square$

Proof of Corollary 1. Apply the proof of Proposition 6 with $\alpha = 1$. $\square$

A2 Some Derivations

A2.1 Private-efficiency condition on wages

I assume that firms are symmetric. I show that condition (8) from Definition 2 is in fact a necessary and sufficient condition for private efficiency of all firm-worker matches. Clearly, since the flow value from unemployment is 0, and since all firms offer the same wage, workers never quit as long as they receive a positive wage $w_t > 0$. I now focus on the firm's optimal behavior, which is detailed in Lemma A1.

LEMMA A1. *Let the marginal revenue $\hat{v}_t(i)$ be defined by*

$$\hat{v}_t(i) \equiv \frac{\partial g}{\partial n(i)}\left((1-s) \cdot n_{t-1}(i), a_t\right).$$

There exist marginal costs $v_t^H(i) > v_t^L(i)$ *such that:*

   (i) if $\hat{v}_t(i) < v_t^L(i)$, *firm i lays off workers;*

   (ii) if $\hat{v}_t(i) \in [v_t^L(i), v_t^H(i)]$, *firm i freezes hiring;*

   (iii) if $\hat{v}_t(i) > v_t^H(i)$, *firm i hires workers.*

Proof. The Lagrangian for firm i's problem, which accounts for possible layoffs, is

$$\mathcal{L} = \mathbb{E}_0 \sum_{t \geq 0} \delta^t \cdot \left\{ g(n_t(i), a_t) - n_t(i) \cdot w(n_t(i), \theta_t, n_t, a_t) \right.$$
$$\left. - \mathbf{1}\{n_t(i) > (1-s) \cdot n_{t-1}(i)\} \cdot \frac{c \cdot a_t}{q(\theta_t)} \cdot [n_t(i) - (1-s) \cdot n_{t-1}(i)] \right\},$$

where $\mathbf{1}\{x\}$ is the indicator function ($\mathbf{1}\{x\} = 1$ if and only if x is true). The firm's problem is a concave maximization problem, so it admits a unique solution determined by the first-order conditions. The highest marginal product of labor that firm i can obtain in period t without laying off workers is

$$\hat{v}_t(i) = \frac{\partial g}{\partial n(i)} \left((1-s) \cdot n_{t-1}(i), a_t \right).$$

I define the following marginal costs:

$$v_t^L(i) \equiv w_t(i) + (1-s) \cdot n_t(i) \cdot \frac{\partial w}{\partial n(i)} ((1-s) \cdot n_t(i), \theta_t, n_t, a_t) - \delta \cdot \mathbb{E}_t \left[\frac{\partial L_{t+1}}{\partial n_t(i)} \right]$$
$$v_t^H(i) \equiv w_t(i) + (1-s) \cdot n_t(i) \cdot \frac{\partial w}{\partial n(i)} ((1-s) \cdot n_t(i), \theta_t, n_t, a_t) + \frac{c \cdot a_t}{q(\theta_t)} - \delta \cdot \mathbb{E}_t \left[\frac{\partial L_{t+1}}{\partial n_t(i)} \right],$$

where I define:

$$L_{t+1} \equiv \sum_{\tau \geq t+1} \delta^{\tau-(t+1)} \left\{ g(n_\tau(i), a_\tau) - n_\tau(i) \cdot w(n_\tau(i), \theta_\tau, n_\tau, a_\tau) \right.$$
$$\left. - \mathbf{1}\{n_\tau(i) > (1-s) \cdot n_{\tau-1}(i)\} \cdot \frac{c \cdot a_\tau}{q(\theta_\tau)} \cdot [n_\tau(i) - (1-s) \cdot n_{\tau-1}(i)] \right\}.$$

Computing $v_t^L(i)$ and $v_t^H(i)$ requires computing $\mathbb{E}_t [\partial L_{t+1}/\partial n_t(i)]$. Let $\mathcal{F}$ be the $\sigma-$algebra generated by future realizations of the stochastic process $\{a_\tau, \tau \geq t+1\}$, taking as given the information set at time t. I partition $\mathcal{F}$ as follows:

$$\mathcal{F} = \mathcal{F}^+ \cup \mathcal{F}^- \cup_{h=1}^{+\infty} \mathcal{F}^h. \tag{A8}$$

$\mathcal{F}^+$ is the subset of future realizations of $\{a_\tau\}$ such that there is hiring next period. $\mathcal{F}^-$ is the subset such that there are layoffs next period. For $h \geq 1$, $\mathcal{F}^h$ is the subset such that there is a hiring freeze for the h next periods. Let $p^+ = \mathbb{P}(\mathcal{F}^+)$, $p^- = \mathbb{P}(\mathcal{F}^-)$, and $p^h = \mathbb{P}(\mathcal{F}^h)$ be the measures of these subsets. Using the law

6

of total probability over this partition,

$$\mathbb{E}_t\left[\frac{\partial L_{t+1}}{\partial n_t(i)}\right] = p^+ \cdot \mathbb{E}_t\left[\frac{\partial L_{t+1}}{\partial n_t(i)}|\mathcal{F}^+\right] + p^- \cdot \mathbb{E}_t\left[\frac{\partial L_{t+1}}{\partial n_t(i)}|\mathcal{F}^-\right] + \sum_{h=1}^{+\infty} p^h \cdot \mathbb{E}_t\left[\frac{\partial L_{t+1}}{\partial n_t(i)}|\mathcal{F}^h\right].$$

It is easy to show that:

$$\mathbb{E}_t\left[\frac{\partial L_{t+1}}{\partial n_t(i)}|\mathcal{F}^+\right] = (1-s)\cdot\mathbb{E}_t\left[\frac{c\cdot a_{t+1}}{q(\theta_{t+1})}|\mathcal{F}^+\right]$$

$$\mathbb{E}_t\left[\frac{\partial L_{t+1}}{\partial n_t(i)}|\mathcal{F}^-\right] = 0$$

$$\mathbb{E}_t\left[\frac{\partial L_{t+1}}{\partial n_t(i)}|\mathcal{F}^h\right] = \mathbb{E}_t\left[\sum_{j=t+1}^{t+h} \delta^{j-(t+1)}\cdot(1-s)^{j-t}\cdot\left\{\frac{\partial g}{\partial n(i)}((1-s)^{j-t}\cdot n_t(i),a_j)\right.\right.$$
$$\left.\left. -(1-s)^{j-t}\cdot n_t(i)\cdot\frac{\partial w}{\partial n(i)}((1-s)^{j-t}\cdot n_t(i),\theta_j,n_j,a_j)-w_j(i)\right\}+\delta^h(1-s)^{h+1}\cdot\frac{c\cdot a_{t+h+1}}{q(\theta_{t+h+1})}|\mathcal{F}^h\right].$$

$v_t^L(i)$ and $v_t^H(i)$ are well defined. They depend on the stochastic process $\{\theta_\tau, \tau \geq t+1\}$ and on employment $(1-s)\cdot n_{t-1}(i)$ at the beginning of period t. I assume that the marginal cost is increasing in $n_t(i)$, and that the marginal product of labor $\partial g/\partial n$ is decreasing in $n_t(i)$ (which is true with constant or diminishing marginal returns to labor in production). $v_t^L(i)$ is the lowest marginal cost that the firm can achieve by keeping all its workforce. This is achieved by freezing hiring. $v_t^H(i) \geq v_t^L(i)$ is the lowest marginal cost that the firm can achieve while recruiting workers. It is achieved by recruiting an infinitely small amount of workers. The optimal decision of the firm is obtained by comparing $v_t^L(i), v_t^h(i)$, and $\hat{v}_t(i)$. The optimal decision of the firm is characterized by the equality of marginal cost and marginal product of labor. If $\hat{v}_t(i) < v_t^L(i)$, firm i lays off workers to increase its marginal product of labor and reduce its marginal cost. Conversely if $\hat{v}_t(i) > v_t^H(i)$, firm i hires workers to reduce its marginal product of labor and increase its marginal cost until both are equal. If $\hat{v}_t(i) \in [v_t^L(i), v_t^H(i)]$, firm i freezes hiring. $\qquad\square$

If a firm freezes hiring in a symmetric environment, all firms do so: $\theta_t = 0$, $c\cdot a_t/q(\theta_t) = 0$, and $v_t^L(i) = v_t^H(i)$ for all i. Thus hiring freezes occur with probability 0. Either all firms recruit, or they all lay off workers. Using Lemma A1 and its proof, we know that in a symmetric environment a necessary and sufficient condition to avoid layoffs in period t is $\hat{v}_t \geq v_t^L$. Moreover, $\mathbb{E}_t[c\cdot a_{t+1}/q(\theta_{t+1})] = \mathbb{E}_t[c\cdot a_{t+1}/q(\theta_{t+1})|\mathcal{F}^+]\cdot p^+$ because $\mathbb{E}_t[c\cdot a_{t+1}/q(\theta_{t+1})|\mathcal{F}^-] = 0$, and $p^h = 0$ for all h, using the partition defined by (A8). Therefore

$$\mathbb{E}_t[\partial L_{t+1}/\partial n_t(i)] = \mathbb{E}_t\left[\frac{c\cdot a_{t+1}}{q(\theta_{t+1})}|\mathcal{F}^+\right]\cdot p^+ = \mathbb{E}_t\left[\frac{c\cdot a_{t+1}}{q(\theta_{t+1})}\right].$$

Accordingly, a necessary and sufficient condition to avoid layoffs is that for all t,

$$w_t \leq \frac{\partial g}{\partial n(i)}((1-s)\cdot n_{t-1},a_t)-(1-s)\cdot n_{t-1}\cdot\frac{\partial w}{\partial n(i)}((1-s)\cdot n_{t-1},0,(1-s)\cdot n_{t-1},a_t)+\delta\cdot(1-s)\cdot\mathbb{E}_t\left[\frac{c\cdot a_{t+1}}{q(\theta_{t+1})}\right].$$

A2.2 Semi-elasticity e_u^w of wages with respect to unemployment

Let $\varepsilon_y^x \equiv d\ln(x)/d\ln(y)$ be the elasticity of x with respect to y and $e_y^x \equiv d\ln(x)/dy$ be the semi-elasticity of x with respect to y. Using various definitions and equilibrium conditions (3) and (14), I compute the following elasticities in steady state:

$$\varepsilon_a^w = \gamma$$

$$e_u^w = \frac{1}{\bar{u}} \cdot \varepsilon_u^w = \frac{1}{\bar{u} \cdot \varepsilon_w^u} = \left[\frac{\bar{u}}{\gamma} \cdot \varepsilon_a^u \right]^{-1}$$

$$\varepsilon_n^u = -\frac{1 - \bar{u}}{\bar{u}}$$

$$\varepsilon_\theta^n = (1 - \eta) \cdot \bar{u}$$

$$\varepsilon_a^\theta = \frac{1 - \gamma}{\eta} \cdot \frac{\omega}{\alpha \cdot \bar{n}^{\alpha - 1} \left[1 + (1 - \alpha) \cdot \frac{1 - \eta}{\eta} \cdot \bar{u} \right] - \omega}$$

$$e_u^w = -\left[\bar{u} \cdot (1 - \bar{u}) \cdot \frac{1 - \gamma}{\gamma} \cdot \frac{1 - \eta}{\eta} \cdot \frac{\omega}{\alpha \cdot \bar{n}^{\alpha - 1} \left[1 + (1 - \alpha) \cdot \frac{1 - \eta}{\eta} \cdot \bar{u} \right] - \omega} \right]^{-1}.$$

A2.3 Wage rigidity in newly created jobs and in existing jobs

In period t, let $w_{t,t}$ be the wage in jobs newly created, and let $w_{\tau,t}$ be the wage in existing jobs created in period $\tau < t$. Consider a world in which wages follow the wage schedule in Assumption 5: for all t and for all $\tau \leq t$

$$w_{\tau,t} = \omega \cdot a_t^\gamma.$$

There is a unique wage prevailing in all jobs (newly created and existing jobs alike). The wage only depends on technology in the current period.

Consider an alternative world in which wages follow an alternative wage schedule. In newly created jobs at time t,

$$w_{t,t} = \omega \cdot a_t^\gamma$$

and in existing jobs created at time $\tau < t$,

$$w_{\tau,t} = w_{t-1} \cdot \left(\frac{a_t}{a_{t-1}} \right)^\zeta.$$

Equivalently, in jobs created at time $\tau \leq t$ wages are given by

$$w_{\tau,t} = \omega \cdot a_\tau^\gamma \cdot a_\tau^{-\zeta} \cdot a_t^\zeta$$

Notice that if $\zeta = \gamma$, both worlds are identical.

The wage schedule in the second world is more realistic as it allows for a different wage rigidity in

newly created jobs and in existing jobs in line with empirical evidence [Pissarides, 2009]. Does the wage rigidity ζ in existing jobs matter for firm's recruiting decisions? Firm's recruiting decisions solely depend on the expected present value of wages paid during the entire duration of a match [Hall and Milgrom, 2008; Pissarides, 2009]. In the first world the expected present value for a job created at t is

$$
\begin{aligned}
W_t^1 &= \sum_{j=0}^{+\infty} \delta^j \cdot (1-s)^j \, \mathbb{E}_t \left[w_{t,t+j} \right] \\
&= \omega \cdot \sum_{j=0}^{+\infty} \delta^j \cdot (1-s)^j \, \mathbb{E}_t \left[a_{t+j}^\gamma \right] \\
&= \omega \cdot \sum_{j=0}^{+\infty} \delta^j \cdot (1-s)^j \left(\mathbb{E}_t \left[a_{t+j} \right] \right)^\gamma \\
&= \omega \cdot \sum_{j=0}^{+\infty} \delta^j \cdot (1-s)^j \left(a_t \right)^\gamma \\
&= \omega \cdot \frac{a_t^\gamma}{1 - \delta \cdot (1-s)}
\end{aligned}
$$

The second line is obtained using the certainty equivalence approximation (valid if γ is close enough to 1 such that the function $x \mapsto x^\gamma$ be linear enough). The third line is obtained by approximating the technology process by a random walk such that $\mathbb{E}_t \left[a_{t+j} \right] = a_t$ for all $j \geq 0$ (empirically, I cannot reject that technology follows a random walk in US data). The elasticity of W_t^1 with respect to a_t is γ.

In the second world the expected present value for a job created at t is

$$
\begin{aligned}
W_t^2 &= \sum_{j=0}^{+\infty} \delta^j \cdot (1-s)^j \, \mathbb{E}_t \left[w_{t,t+j} \right] \\
&= \omega \cdot a_t^\gamma \cdot \sum_{j=0}^{+\infty} \delta^j \cdot (1-s)^j \cdot \frac{1}{a_t^\zeta} \mathbb{E}_t \left[a_{t+j}^\zeta \right] \\
&= \omega \cdot a_t^\gamma \cdot \sum_{j=0}^{+\infty} \delta^j \cdot (1-s)^j \cdot \frac{1}{a_t^\zeta} \left(\mathbb{E}_t \left[a_{t+j} \right] \right)^\zeta \\
&= \omega \cdot \frac{a_t^\gamma}{1 - \delta \cdot (1-s)}
\end{aligned}
$$

This result is obtained using the same approximations as above (certainty equivalence and random walk). Crucially, $W_t^1 = W_t^2$. The wage schedule in Assumption 5 leads to the same recruiting behavior as a realistic two-tier wage schedule in which wage flexibility is different for newly created and existing jobs, as long as both schedules have the same wage flexibility for newly created jobs (measured by the elasticity of wages in newly created jobs with respect to technology). Furthermore the elasticity of W_t^2 with respect to a_t is γ, independent of ζ. It is critical to estimate the flexibility of wages in newly created jobs. The reason is that only this flexibility matters to explain the recruiting behavior of firms over the business cycle, even if wages in existing jobs have a different flexibility from wages in newly created jobs.

9

A2.4   Log-linearized model

I first characterize the steady state of the model. I then describe the log-linearized equilibrium conditions around the steady state. The symmetric steady-state equilibrium $\{\bar{c}, \bar{n}, \bar{y}, \bar{h}, \bar{\theta}, \bar{u}, \bar{w}\}$ is characterized by

$$\bar{u} = \frac{s}{s + (1-s) \cdot f(\bar{\theta})}$$

$$\bar{n} = \frac{1 - \bar{u}}{1 - s}$$

$$\bar{h} = s \cdot \bar{n}$$

$$\bar{y} = \bar{a} \cdot \bar{n}^{\alpha}$$

$$\bar{c} = \bar{y} - \frac{c \cdot \bar{a}}{q(\bar{\theta})} \cdot \bar{h}$$

$$\bar{w} = \omega$$

$$0 = \alpha \cdot \bar{n}^{\alpha - 1} - \bar{w} - [1 - \delta \cdot (1-s)] \frac{c \cdot \bar{a}}{q(\bar{\theta})}$$

$$\bar{a} = 1$$

In steady state the components of unemployment satisfy

$$\bar{u}^R = 1 - (\alpha/\omega)^{1/(1-\alpha)}$$

$$\bar{u}^F = \bar{u} - \bar{u}^R.$$

Let $\check{x}_t \equiv d\ln(x_t)$ denotes the logarithmic deviation of variable x_t. The equilibrium is described by the following system of log-linearized equations:

- Definition of labor market tightness:

$$(1 - \eta) \cdot \check{\theta}_t = \check{h}_t - \check{u}_{t-1}$$

- Definition of unemployment:

$$\check{u}_t + \frac{1 - \bar{u}}{\bar{u}} \cdot \check{n}_{t-1} = 0$$

- Law of motion of employment:

$$\check{n}_t = (1-s) \cdot \check{n}_{t-1} + s \cdot \check{h}_t$$

- Resource constraint in the economy (all production is either consumed or allocated to recruiting):

$$\check{y}_t = (1 - s_1) \cdot \check{c}_t + s_1 \cdot \left(\check{h}_t + \eta \cdot \check{\theta}_t + \check{a}_t\right),$$

with $s_1 = \dfrac{c}{q(\bar{\theta})} \cdot s \cdot \bar{n}^{1-\alpha}$.

- Production constraint:

$$\check{y}_t = \check{a}_t + \alpha \cdot \check{n}_t$$

- Wage rule:

$$\check{w}_t = \gamma \cdot \check{a}_t$$

- Firm's optimality condition:

$$-\check{a}_t + (1-\alpha) \cdot \check{n}_t + s_2 \cdot \check{w}_t + s_3 \cdot \left(\eta \cdot \check{\theta}_t + \check{a}_t\right) + (1 - s_2 - s_3) \cdot \mathbb{E}_t \left[\eta \cdot \check{\theta}_{t+1} + \check{a}_{t+1}\right] = 0$$

with $s_2 = \overline{w} \cdot \frac{1}{\alpha \cdot \overline{a}} \cdot \overline{n}^{1-\alpha}$ and $s_3 = \frac{c}{q(\overline{\theta})} \cdot \frac{1}{\alpha} \cdot \overline{n}^{1-\alpha}$.

- Productivity shock:

$$\check{a}_t = \rho \cdot \check{a}_{t-1} + z_t$$

The components of unemployment are described by the following log-linear equations:

$$\check{u}_t^R = -\frac{1-\gamma}{1-\alpha} \cdot \frac{1-\overline{u}^R}{\overline{u}^R} \cdot \check{a}_{t-1}$$

$$\check{u}_t = \frac{\overline{u} - \overline{u}^R}{\overline{u}} \cdot \check{u}_t^F + \frac{\overline{u}^R}{\overline{u}} \cdot \check{u}_t^R.$$

A3 Other Decompositions of Unemployment

In this section I approach the decomposition of unemployment presented in Section IV.D from another angle. I decompose actual US unemployment into rationing and frictional series instead of decomposing model-generated unemployment. To do so, I back out from observable series in US data the series of unobserved shocks necessary for the model to match the data exactly.

As in Shimer [2005] and Pissarides [2009], I approximate the fully dynamic model at time t when the realization of technology is $a_t = a$ by the equilibrium of the static model with technology a. In particular, I assume that labor market tightness θ_t is related to employment n_t at any time by the Beveridge curve (3). This approximation is motivated by the observation that the labor market rapidly converges to an equilibrium in which inflows to and outflows from employment are balanced because rates of inflow to and outflow from unemployment are large, while technology shows a lot of persistence [Hall, 2005; Pissarides, 1986, 2009; Rotemberg, 2008; Shimer, 2007]. Employment n_t is determined by the firm's optimality condition (14). As pointed out by Pissarides [2009], abstracting from stochastic fluctuations in technology does not reduce the realism of the model much because technology is quite persistent in the data.

In this section, I explore the possibility that data are generated by three types of shocks:

- technology shocks $\{a_t\}$—this shock is the one considered in the search-and-matching literature and in the text;

- wage shocks $\{\omega_t\}$ that influence the wage level $w_t = \omega_t \cdot a_t^\gamma$—historical studies suggest that this shock may have played a large role during the Great Depression;

- matching shocks $\{\mu_t\}$ that influence the matching function $h(u_t, v_t) = \mu_t \cdot u_t^\eta \cdot v_t^{1-\eta}$—the shift of the Beveridge curve in 2008–2010 in the US suggests that this shock may have played a role in the 2008–2010 recession.

Under the realization of shocks $\{a_t, \omega_t, \mu_t\}$, employment n_t, labor market tightness θ_t, and output y_t are given by a system of three equations:

$$\left[\frac{1}{\mu_t} \cdot \frac{s \cdot n_t}{1 - (1-s) \cdot n_t}\right]^{1/(1-\eta)} = \theta_t$$

$$\alpha \cdot n_t^{\alpha-1} - \omega_t \cdot a_t^{\gamma-1} = [1 - (1-s) \cdot \delta] \cdot \frac{c}{\mu_t} \cdot \theta_t^\eta$$

$$y_t = a_t \cdot n_t^\alpha$$

which defines implicit functions for employment, output, and labor market tightness:

$$n_t = n(a_t, \omega_t, \mu_t) \tag{A9}$$

$$y_t = y(n_t, a_t) \tag{A10}$$

$$\theta_t = \theta(n_t, \mu_t). \tag{A11}$$

With the labor market in steady state, $n_t = n_{t-1}$ and unemployment is related to employment by $u_t = 1 - (1-s) \cdot n_t$. Hence equations (A9)-(A11) define implicit relationships between three observable variables $\{u_t, y_t, \theta_t\}$ and three unobservable shocks $\{a_t, \omega_t, \mu_t\}$, which we can write

$$u_t = u(a_t, \omega_t, \mu_t). \tag{A12}$$

$$y_t = y(u_t, a_t). \tag{A13}$$

$$\theta_t = \theta(u_t, \mu_t). \tag{A14}$$

As technology shocks, matching shocks, and wage shocks are not directly observable, I exploit the structure imposed by the model to back out the realizations of shocks $\{a_t^*, \omega_t^*, \mu_t^*\}$ that generated the observable series $\{\hat{u}_t, \hat{\theta}_t, \hat{y}_t\}$ in US data. The observable series used are described in details in Section IV. I use the shock series to construct series for frictional unemployment and rationing unemployment:

$$u_t^R = \max\left\{0, 1 - \left(\frac{\alpha}{\omega_t}\right)^{\frac{1}{1-\alpha}} \cdot a_t^{\frac{1-\gamma}{1-\alpha}}\right\}, \tag{A15}$$

$$u_t^F = u_t - u_t^R, \tag{A16}$$

which have the property that their sum matches exactly observed unemployment.

A3.1 Technology shock

I first assume that there is only a technology shock, and $\omega_t = \omega$ and $\mu_t = \mu$ for all t, where ω and μ are calibrated in Table 1. From observed employment $\{\hat{u}_t\}$, I can construct the implied technology series $\{a_t^*\}$ using (A12). For all t, a_t^* solves

$$\hat{u}_t = u(a_t^*, \omega, \mu).$$

Then, I construct rationing and frictional unemployment rates from $\{a_t^*\}$ and equations (A15) and (A16).

The decomposition is shown on the top panel in Figure A3, and is quantitatively similar to that presented in Figure (4). Current events illustrate how the composition of unemployment drastically changes over the business cycle. In 2006:Q4, detrended unemployment was at 4.9%, frictional unemployment was 4.3%, and rationing unemployment was only 0.6%. In 2008:Q2, unemployment was at 5.8%, of which 3.6% was frictional unemployment and 2.2% was rationing unemployment. Finally in 2009:Q2, unemployment reached 9.9%, frictional unemployment fell to 2.3%, and rationing unemployment increased to 7.6%.

A3.2 Technology and wage shocks

I assume that unemployment fluctuations are driven by technology and wage shocks $\{a_t, \omega_t\}$. I assume that $\mu_t = \mu$ for all t, where μ is calibrated in Table 1. From observed unemployment $\{\hat{u}_t\}$ and observed output $\{\hat{y}_t\}$, I can retrieve the series of technology and wage shocks $\{a_t^*, \omega_t^*\}$ that generated the data using (A12) and (A13): for all t, a_t^* and $\omega*_t$ solve

$$\hat{u}_t = u(a_t^*, \omega_t^*, \mu)$$
$$\hat{y}_t = y(\hat{u}_t, a_t^*).$$

As showed in Figure A1, $\{a_t^*\}$ is similar to the total factor productivity series constructed by Fernald [2009], which accounts for variable capital utilization and labor hoarding. The similarity suggests that the model offers a good description of labor market fluctuations at business cycle frequency.

I construct rationing and frictional unemployment rates from $\{a_t^*, \omega_t^*\}$ and equations (A15) and (A16). The decomposition is not shown as it is exactly similar to the one on the top panel. In fact, for an observed unemployment series, I can infer implied employment $\hat{n}_t = (1 - \hat{u}_t)/(1 - s))$ and implied tightness $\hat{\theta}_t = \theta(\hat{u}_t, \mu)$. The estimated ratio $(w_t/a_t)^*$ always satisfies

$$(w_t/a_t)^* = \alpha \cdot \hat{n}_t^\alpha - [1 - (1 - s) \cdot \delta] \cdot \frac{c}{q(\hat{\theta}_t)}. \tag{A17}$$

For a given observed unemployment $\hat{u}_t$, frictional and rationing unemployment depend only on $(w_t/a_t)^*$
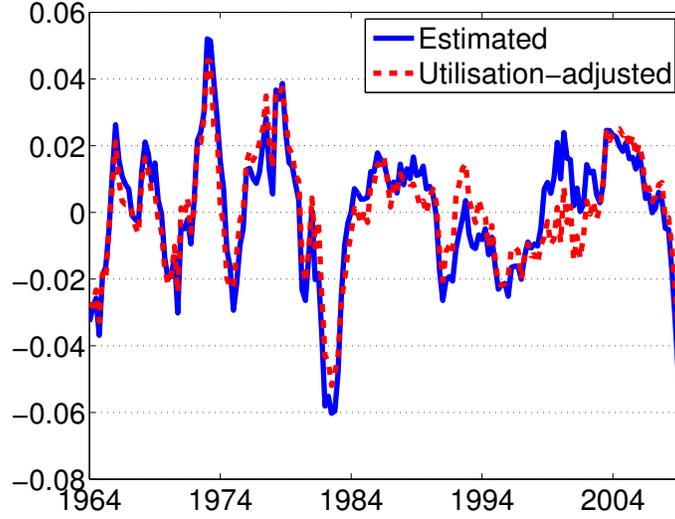
Figure A1: Estimated technology and utilization-adjusted technology in US data, 1964–2009

Notes: This graph compares the quarterly, utilization-adjusted TFP series constructed by Fernald [2009], to the quarterly technology series $\{a_t^*\}$ estimated in the text from observed output and unemployment data by solving $\hat{y}_t = y(\hat{u}_t, a_t^*)$ for a_t^*. The time period is 1964:Q1–2009:Q2. These quarterly series are reported after detrending the log of the series using an HP filter with smoothing parameter 10^5.

through

$$u_t^R = \max\left\{ 0, 1 - \left[\frac{1}{\alpha} \cdot (w_t/a_t)^*\right]^{-1/(1-\alpha)} \right\}.$$

$$u_t^F = \hat{u}_t - u_t^R.$$

To conclude, if only technology and wage shocks hit the economy, the decomposition of observed unemployment does not depend on the specific realizations of the technology and wage shocks, as the ratio w_t/a_t is bound to satisfy (A17).

A3.3 Technology, matching, and wage shocks

I now assume that unemployment fluctuations are driven by technology, matching, and wage shocks. From observed unemployment $\{\hat{u}_t\}$, observed output $\{\hat{y}_t\}$, and observed labor market tightness $\{\hat{\theta}_t\}$, I can retrieve the series of technology, wage, and matching shocks $\{a_t^*, \omega_t^*, \mu_t^*\}$ that generated the data using (A12), (A13), and (A14). For all t, a_t^*, ω_t^*, μ_t^* solve

$$\hat{u}_t = u(a_t^*, \omega_t^*, \mu)$$

$$\hat{y}_t = y(\hat{u}_t, a_t^*)$$

$$\hat{\theta}_t = \theta(\hat{u}_t, \mu_t^*)$$

14
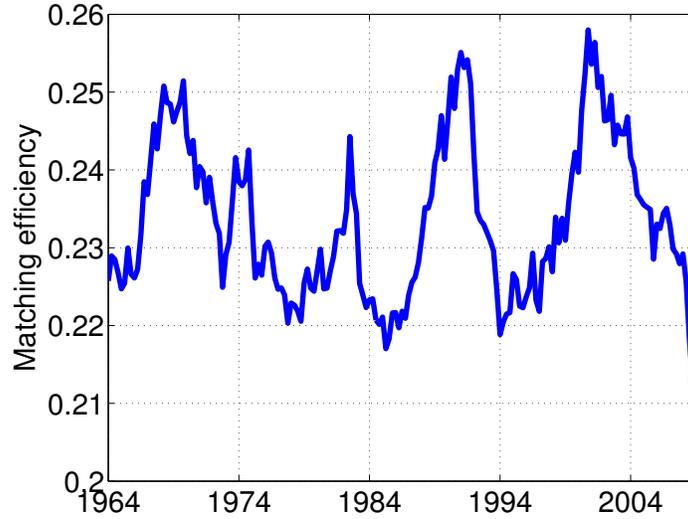
Figure A2: Matching efficiency in US data, 1964–2009

Notes: The graph is obtained by solving $\hat{\theta}_t = \theta(\hat{u}_t, \mu_t^*)$ for μ_t^* for all t, given observed unemployment and labor market tightness $\{\hat{u}_t, \hat{\theta}_t\}$. The time period is 1964:Q1–2009:Q2.

The estimated matching shocks $\{\mu_t^*\}$ are displayed in Figure A2. Matching efficiency does fluctuate, and as pointed out by various observers of the labor market, the efficiency of matching has fallen drastically in the current 2008–2010 recession (adverse shift of the Beveridge curve). In spite of these fluctuations, the quantitative results are scarcely affected. I construct rationing and frictional unemployment rates from observed unemployment $\{\hat{u}_t\}$, estimated shocks $\{a_t^*, \omega_t^*\}$, and equations (A15) and (A16). The decomposition is shown on the bottom panel in Figure A3. It is similar to the one in the top panel. Hence, shocks to matching efficiency do not modify quantitatively the decomposition of unemployment. It is worth noting that, compared to the earlier results, the drop in frictional unemployment in the current recession is slightly milder because the increase in unemployment is caused in part by a reduction in matching efficiency. More precisely, in 2006:Q4, actual detrended unemployment was at 4.9%, frictional unemployment was 4.2%, and rationing unemployment was only 0.7%. In 2008:Q2, unemployment was at 5.8%, of which 3.8% was frictional unemployment and 2.0% was rationing unemployment. Finally, in 2009:Q2, unemployment reached 9.9%, frictional unemployment fell to 2.8%, and rationing unemployment increased to 7.1%.

A4 Sensitivity Analysis of Quantitative Results

A4.1 Calibration of recruiting cost

There is a relatively wide range of admissible values for the recruiting cost c. Reasonable calibrations of c would be in the range $[0.098 \cdot \omega, 0.54 \cdot \omega]$. In the text I picked the mid-point of this range: $c = 0.32 \cdot \omega$. This estimate has the advantage of being well within the range of estimates used in the literature. Using the average unemployment rate and labor market tightness in JOLTS, I find that $c = 0.32$ corresponds to

15
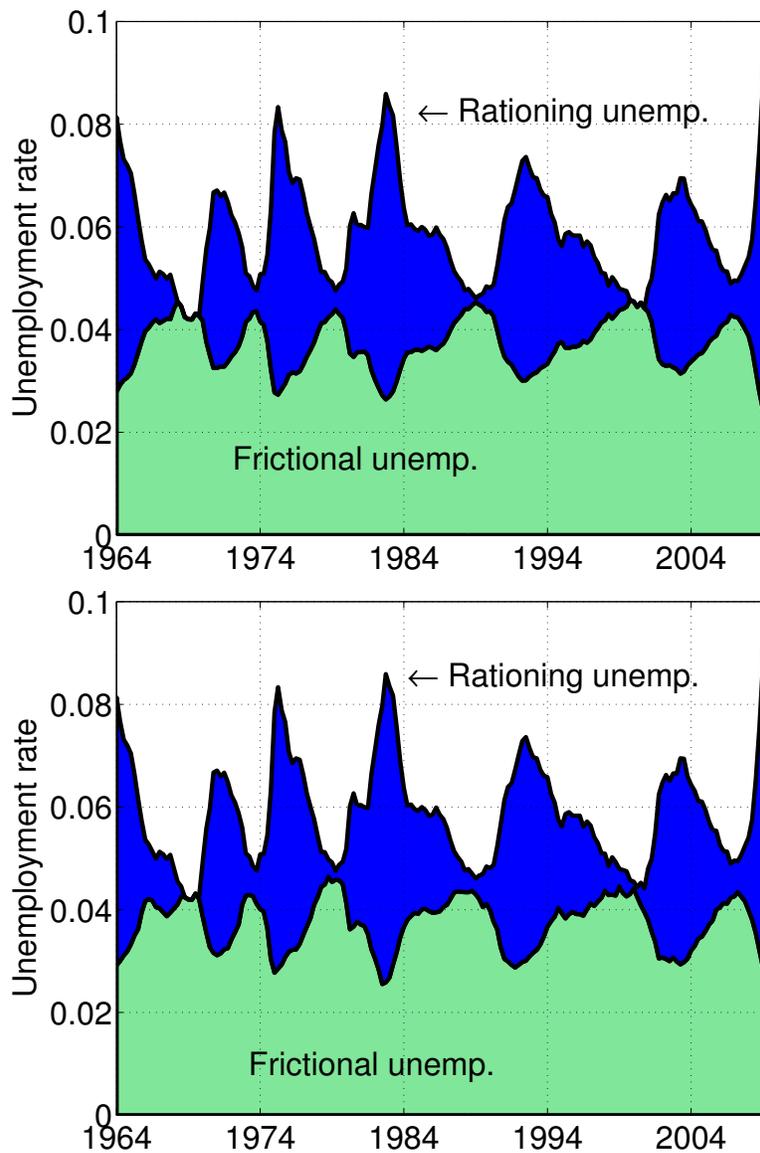
Figure A3: Decomposition of actual US unemployment, 1964–2009

Notes: The graph decomposes actual unemployment, which is the quarterly average of seasonally-adjusted monthly series constructed by the BLS from the CPS. Actual unemployment is detrended with an HP filter with smoothing parameter 10^5. The time period is 1964:Q1–2009:Q2. The top panel is obtained by inferring the series of technology shocks needed to match actual unemployment, or equivalently, by inferring the series of wage shocks and technology shocks needed to match both actual unemployment and output. The bottom panel is obtained by inferring the series of wage shocks, technology shocks, and matching shocks needed to match actual unemployment, output, and labor market tightness.

16

0.89% of the total wage bill being spent on recruiting. This is in line with a long list of papers who use a ratio of adjustment costs to output of 1% [Andolfatto, 1996; Barnichon, 2010; Blanchard and Galí, 2010; Gertler and Trigari, 2009; Thomas, 2008]. Since I estimate recruiting cost to be 0.32 of a worker's wage, my paper is also in line with influential papers: for example, Shimer [2005] picks vacancy-posting cost to be 0.213 of a worker's wage, Elsby and Michaels [2008] picks 0.27, Pissarides [2009] picks 0.357, and Hall and Milgrom [2008] picks 0.433 .

In this section I examine how the quantitative properties of the model change with a low ($c = 0.0.98 \cdot \omega$) and a high ($c = 0.54 \cdot \omega$) recruiting cost. The calibration strategy in Section IV implies that calibrating the recruiting cost differently affects the production-function parameter α and the steady-state wage ω. Under the low-cost calibration, $c = 0.066$, $\alpha = 0.662$, and $\omega = 0.671$. Under the high-cost calibration, $c = 0.362$, $\alpha = 0.671$, and $\omega = 0.671$.

Table A1 reports the simulated moments under the alternative calibrations. The main difference with simulated moments reported in Table 3 is that standard deviations for unemployment, vacancy, and labor market tightness are lower in the high-cost model, and are higher in the low-cost model. This was expected as in the high-cost model recruiting costs, which are countercyclical and therefore dampen fluctuations in technology and reduce the amplitude of fluctuations, play a more important buffer role than in the calibration in the text (the opposite mechanism is at work in the low-cost model). The amplification of technology shocks on the labor market under these alternative specifications is modified accordingly:

- In the high-cost model a 1-percent decrease in technology increases unemployment by 4.5%, reduces vacancy by 5.0%, and reduces labor market tightness by 9.5%.

- In the low-cost model a 1-percent decrease in technology increases unemployment by 9.2%, reduces vacancy by 10.4%, and reduces labor market tightness by 19.6%.

- This compares to US data, where a 1-percent decrease in technology increases unemployment by 4.2%, reduces vacancy by 4.3% and reduces labor market tightness by 8.6%.

The sensitivity analysis confirms that any of these calibrations allows the model to match the amplification of technology observed in the data. Moreover the behavior of the wage, correlations, and autocorrelations are virtually identical to those reported in the text, and the conclusions are unaffected.

The second question is how decomposition of unemployment is affected by these alternative calibrations. Clearly the steady-state decomposition is different as higher recruiting costs imply higher frictional unemployment. In fact, in steady state:

- if $c = 0.362$: $u = 5.8\%$, $u^R = 0.1\%$, $u^F = 5.7\%$;

- if $c = 0.066$: $u = 5.8\%$, $u^R = 4.1\%$, $u^F = 1.7\%$.

Figure A4 presents the dynamic response of the model. The responses of labor market variables are not affected by the new calibration beyond what was already noted with simulated moments (the amplification of technology shocks and the volatility of labor market variables are smaller in a model with larger recruiting

Table A1: Simulated moments with technology shocks and various recruiting costs

	Low recruiting cost: $c = 0.066$					
	u	v	θ	w	y	a
Standard Deviation	0.141	0.176	0.308	0.011	0.021	0.015
Autocorrelation	0.897	0.750	0.853	0.821	0.844	0.821
Correlation	1	-0.887	-0.964	-0.981	-0.989	-0.981
	–	1	0.978	0.888	0.874	0.888
	–	–	1	0.956	0.951	0.956
	–	–	–	1	0.998	1.000
	–	–	–	–	1	0.998
	–	–	–	–	–	1
	High recruiting cost: $c = 0.362$					
	u	v	θ	w	y	a
Standard Deviation	0.068	0.084	0.148	0.011	0.018	0.015
Autocorrelation	0.906	0.778	0.869	0.826	0.842	0.826
Correlation	1	-0.899	-0.968	-0.979	-0.985	-0.979
	–	1	0.980	0.908	0.901	0.908
	–	–	1	0.964	0.962	0.964
	–	–	–	1	0.999	1.000
	–	–	–	–	1	0.999
	–	–	–	–	–	1

Notes: Results from simulating the log-linearized model with stochastic technology under various calibrations of the recruiting cost c. All variables are reported in log as deviations from an HP trend with smoothing parameter 10^5.
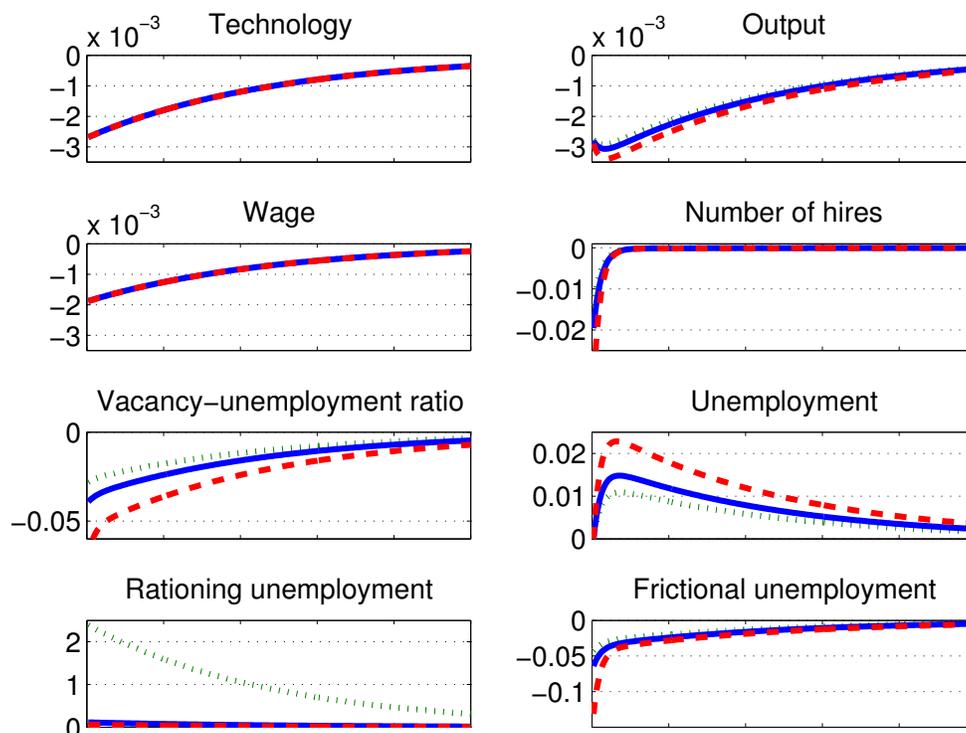
Figure A4: IRFs to a negative technology shock under different calibrations of recruiting cost c

Notes: Impulse response functions (IRFs) represent the logarithmic deviation from steady state for each variable. IRFs are obtained by imposing a negative technology shock of $-\sigma = -0.0027$ to the log-linear model. The time period displayed on the x-axis is 250 weeks. The solid, blue line is under standard parameterization of $c = 0.215$. The dashed, red line is under low-cost parameterization of $c = 0.066$. The dotted, green line is under high-cost parameterization of $c = 0.362$.

cost c). Qualitatively, the dynamic response of unemployment and its components remains the same: both unemployment and rationing unemployment increase after a negative technology shock, while frictional unemployment decreases. The impulse response functions, however, are not easily interpretable because the steady-state values are so different (especially for rationing unemployment).

I repeat the historical decomposition of unemployment under the two alternative calibrations of recruiting cost. Figure A5 shows the historical decomposition of unemployment for low recruiting cost ($c = 0.066$). Frictional unemployment is always lower under this calibration: it remains below 3.5% at all times. As a consequence, the amplitude of fluctuations in frictional unemployment is smaller. Since the amplitude of fluctuations in total unemployment is larger and the amplitude of fluctuations in frictional unemployment is smaller, the fluctuations of rationing unemployment have much larger amplitude. Figure A6 shows the historical decomposition of unemployment for high recruiting cost ($c = 0.362$). It confirms that frictional unemployment is always higher under this calibration. When unemployment is below 5.5%, it is only frictional. Frictional unemployment only falls below 3% in the deep recessions of 1981–1983 and 2008–2009.
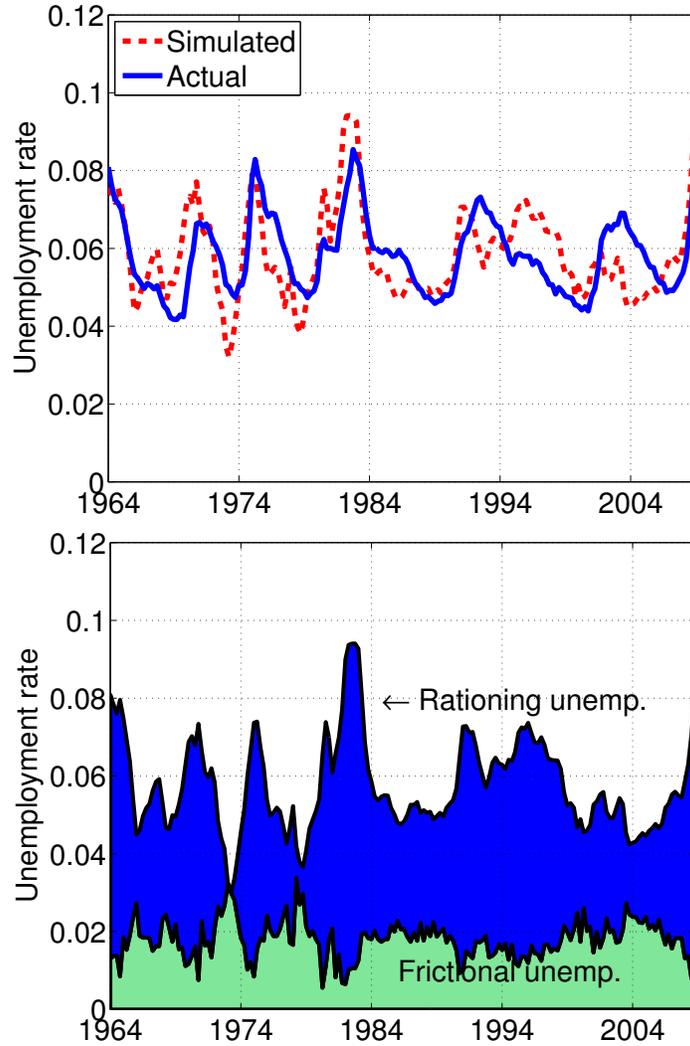
Figure A5: Decomposition of simulated US unemployment, 1964–2009, with low recruiting cost $c = 0.066$.

Notes: Actual unemployment rate is quarterly average of monthly series constructed by the BLS from the CPS. Simulated unemployment rate is generated when the model is stimulated by the quarterly technology series constructed from BLS output and employment data. Actual technology and unemployment are seasonally adjusted. All series are detrended with a HP filter with smoothing parameter 10^5. The time period is 1964:Q1–2009:Q2. I solve the (nonlinear) model with the Fair and Taylor [1983] shooting algorithm. The top graph compares actual and simulated unemployment. The bottom graph decomposes simulated unemployment into frictional unemployment and rationing unemployment.

Figure A6: Decomposition of simulated US unemployment, 1964–2009, with high recruiting cost $c = 0.362$.

Notes: Actual unemployment rate is quarterly average of monthly series constructed by the BLS from the CPS. Simulated unemployment rate is generated when the model is stimulated by the quarterly technology series constructed from BLS output and employment data. Actual technology and unemployment are seasonally adjusted. All series are detrended with a HP filter with smoothing parameter 10^5. The time period is 1964:Q1–2009:Q2. I solve the (nonlinear) model with the Fair and Taylor [1983] shooting algorithm. The top graph compares actual and simulated unemployment. The bottom graph decomposes simulated unemployment into frictional unemployment and rationing unemployment.

A4.2   Specification of recruiting cost

In the text I assume, as in Pissarides [2000], that the per-period cost of opening a vacancy is $c \cdot a_t$. I now assume that the per-period cost of opening a vacancy is c, constant and independent of technology. I assume no randomness at the firm level: a firm fills a job with certainty by opening $1/q(\theta_t)$ vacancies and spending $c/q(\theta_t)$. The modification only affects the firm's optimality condition, which becomes

$$\frac{\partial g}{\partial n(i)}(n_t(i), a_t) = w_t(i) + \frac{c}{q(\theta_t)} + n_t(i) \cdot \frac{\partial w}{\partial n(i)}(n_t(i), \theta_t, n_t, a_t) - \delta \cdot (1 - s) \cdot \mathbb{E}_t \left[\frac{c}{q(\theta_{t+1})} \right]$$

This modification does not affect the theoretical results of the paper. Clearly, Proposition 1 (showing full employment in the canonical model when $c \to 0$), Proposition 2 (showing full employment in the model with diminishing returns when $c \to 0$), Proposition 3 (showing full employment in the model with wage rigidity when $c \to 0$), and Proposition 4 (proving the existence of of job rationing under wage rigidity and diminishing marginal returns to labor) remain valid under this alternative specification of the recruiting cost. Lemma 3 (giving a sufficient condition on wage flexibility γ such that inefficient separations do not occur) also remains valid because the lower bound on γ provided by the lemma is independent from c.

Importantly, Proposition 5 (proving the cyclicality of frictional unemployment and rationing unemployment) remains valid under any reasonable calibration. The firm's optimality (14), modified to account for the new specification of the recruiting cost, still implies that $dn/da > 0$. The expression for rationing unemployment remains the same so that $\partial n^R/\partial a < 0$. Differentiating the firm's optimality (14) with respect to a yields

$$\left\{ \alpha \cdot (1 - \alpha) \cdot (2 - \alpha) \cdot \int_0^{n^F} \frac{\partial n^R}{\partial a} \cdot \left(n^R - x\right)^{\alpha - 3} dx \right\} + \left\{ \alpha \cdot (1 - \alpha) \cdot \frac{\partial n^F}{\partial a} \cdot n^{\alpha - 2} \right\} = [1 - \delta \cdot (1 - s)] \cdot \frac{d}{da} \left[\frac{c/a}{q(\theta)} \right].$$

For Proposition 5 to remain valid, we need to prove that

$$\frac{d}{da} \left[\frac{c/a}{q(\theta)} \right] \geq 0. \tag{A18}$$

It is easy to show that

$$\text{sign}\left(\frac{d}{da} \left[\frac{c/a}{q(\theta)} \right] \right) = \text{sign}\left(\eta \cdot \varepsilon_a^\theta - 1 \right), \tag{A19}$$

where $\varepsilon_a^\theta \equiv (a/\theta) \cdot d\theta/da$ is the elasticity of labor market tightness θ with respect to a. We need to determine

ε_a^θ. From equilibrium equations (3) and (14),

$$\varepsilon_\theta^n = (1 - \eta) \cdot u$$

$$-\alpha \cdot (1 - \alpha) \cdot n^{\alpha-2} \cdot \frac{\partial n}{\partial \theta} \cdot \frac{\partial \theta}{\partial a} = (\gamma - 1) \cdot \omega \cdot a^{\gamma-2} + [1 - (1 - s) \cdot \delta] \cdot \frac{c/a}{q(\theta) \cdot \theta} \cdot \eta \cdot \frac{\partial \theta}{\partial a} - [1 - (1 - s) \cdot \delta] \cdot \frac{c/a}{q(\theta)} \cdot \frac{1}{a}$$

$$(1 - \gamma) \cdot \omega \cdot a^\gamma + [1 - (1 - s) \cdot \delta] \cdot \frac{c}{q(\theta)} = \left[\alpha \cdot (1 - \alpha) \cdot a \cdot n^{\alpha-1} \cdot \varepsilon_\theta^n + [1 - (1 - s) \cdot \delta] \cdot \frac{c}{q(\theta)} \cdot \eta \right] \cdot \varepsilon_a^\theta$$

$$\eta \cdot \varepsilon_a^\theta = \frac{(1 - \gamma) \cdot w + [1 - (1 - s) \cdot \delta] \cdot c/q(\theta)}{(1 - \alpha) \cdot \frac{1 - \eta}{\eta} \cdot u \cdot [w + [1 - (1 - s) \cdot \delta] \cdot c/q(\theta)] + [1 - (1 - s) \cdot \delta] \cdot c/q(\theta)}$$

To prove that condition (A19) is true, we need to show that

$$(1 - \gamma) \geq (1 - \alpha) \cdot \frac{1 - \eta}{\eta} \cdot u \cdot \left(1 + [1 - (1 - s) \cdot \delta] \cdot \frac{c/q(\theta)}{w} \right)$$

This condition ensures that the decrease in θ in a recession is large enough to offset the increase in the normalized recruiting cost c/a when a falls. This condition is necessary because we need recruiting to become cheaper in recessions. The condition does hold for any parameter value: for instance, it cannot hold if $\gamma = 1$. But it is valid for a large range of values. It is accepted in the literature that $\frac{[1-(1-s)\delta]c/q(\theta)}{w} \leq 0.01$: the recruiting cost per worker is no more than 1% of the wage bill [Blanchard and Galí, 2010; Gertler and Trigari, 2009; Thomas, 2008]. Hence the condition simplifies to

$$(1 - \gamma) \geq (1 - \alpha) \cdot \frac{1 - \eta}{\eta} \cdot u.$$

We have $1 - \alpha \leq 1$. We know from Petrongolo and Pissarides [2001] that $\eta \in [0.5, 0.7]$ such that $\eta/(1 - \eta) \leq 1$. Normally, $u \leq 0.1$. This implies that as long as $\gamma \leq 0.9$, condition (A18) is true and our result is valid. In other words, even a small amount of wage rigidity is sufficient for our result to hold.

The question I now examine is whether the alternative specification modifies the quantitative properties of the model with job rationing. I proceed as in the text to compute simulated moments (presented in Table A2) and impulse response functions (presented in Figure A7).

Comparing Table A2 with Table 3, it appears that the quantitative properties of the model are barely affected by this alternative specification. Unemployment, vacancy, and labor market tightness are only slightly more volatile because (i) recruiting costs are slightly higher in recessions ($c \cdot a_t < c$ when $a_t < 1$), which increases unemployment above its level in the model in the text; and (ii) recruiting cost is slightly lower in expansions ($c \cdot a_t > c$ when $a_t > 1$), which reduces unemployment below its level in the text. Technology, however, only varies in the $[0.95, 1.05]$ range in the simulations so $c \cdot a \in [0.205, 0.225]$ whereas $c = 0.215$. The differences in recruiting cost are minor, so the discrepancies in the two sets of simulated moments are minimal.

The IRFs in Figure A7 confirm that the quantitative properties of the model under the two alternative specifications are virtually identical. All the IRFs are superposed but for the IRF of unemployment, which

Table A2: Simulated moments with constant per-period vacancy-posting cost under technology shocks

	u	v	θ	w	y	a
Standard Deviation	0.102	0.126	0.222	0.011	0.020	0.016
Autocorrelation	0.906	0.776	0.867	0.828	0.849	0.828
	1	-0.897	-0.968	-0.980	-0.987	-0.980
	–	1	0.979	0.907	0.896	0.907
	–	–	1	0.964	0.962	0.964
Correlation	–	–	–	1	0.999	1.000
	–	–	–	–	1	0.999
	–	–	–	–	–	1

Notes: Results from simulating the log-linearized model with constant per-period vacancy-posting cost c (instead of $c \cdot a_t$ in the text) under stochastic technology. All variables are reported in log as deviations from an HP trend with smoothing parameter 10^5.

confirms that unemployment is slightly less volatile with the specification in the text (vacancy-posting cost of $c \cdot a_t$) than with this alternative specification (vacancy-posting cost of c). This observation implies that if the model in the text can match the amplification of technology shocks observed in the data, so could this alternative model.

A4.3   Gradual wage adjustment

In view of ethnographic evidence it is possible that wage rigidity, in contrast to the model presented, induces gradual changes in wages in response to shocks. While a gradual change in wages would not modify the theoretical results of the paper (the results are based on comparative statics), it may be important for unemployment dynamics. Gertler and Trigari [2009], Gertler et al. [2008], and Sala et al. [2008] study the impact of staggered wage setting on unemployment dynamics. For tractability, I assume instead that current wage w_t is an average of the past wage w_{t-1} and the wage schedule used in the text:

$$w_t = \left[\omega \cdot a_t^\gamma \right]^\zeta \cdot \left[w_{t-1} \right]^{1-\zeta}. \tag{A20}$$

ζ is the weight placed on the wage schedule that is targeted. If $\zeta = 1$, we are in the case studied in the text. If $\zeta = 0$, wages are completely rigid. In a static environment in which $a_t = a$ for all t, the wage equals the wage schedule used in the text.

To study the dynamics of the model under this alternative assumption, I log-linearize the model. The log-linearized system is the same as in the text, but for the equation giving the log-linearized wage:

$$\check{w}_t = \gamma \cdot \zeta \cdot \check{a}_t + (1 - \zeta) \cdot \check{w}_{t-1}.$$

I calibrate $\zeta = 1.6\%$ to match the correlation $\hat{\rho}(w, a) = 0.646$ of wages and technology in US data. The calibration of all other parameters remains the same as in the text. Table A3 reports the simulated moments.
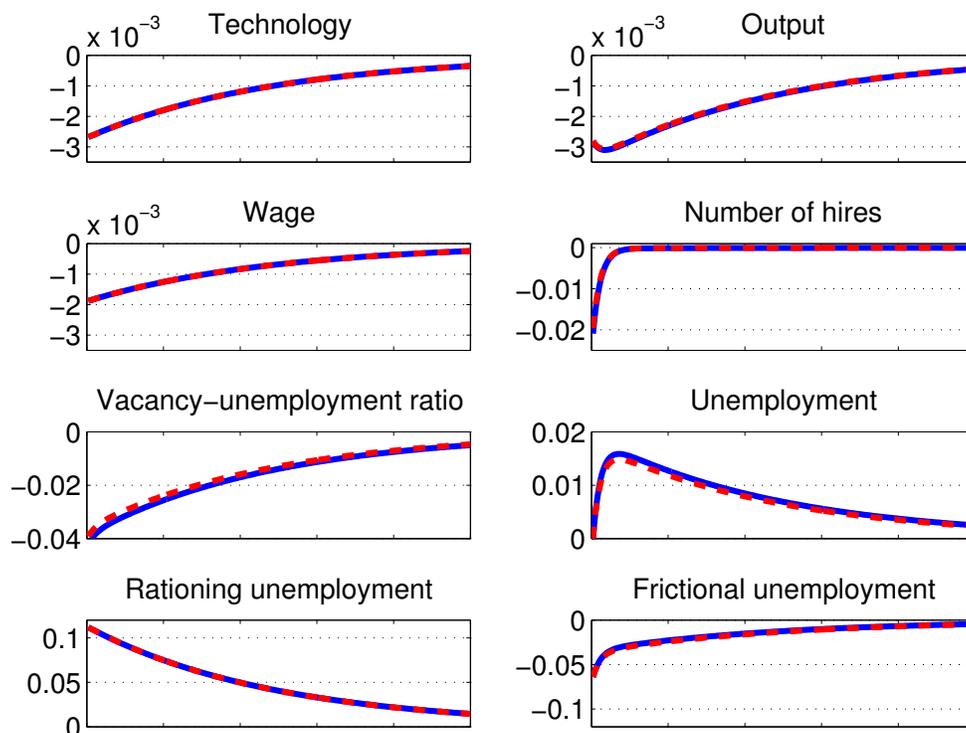
Figure A7: IRFs to a negative technology shock with constant per-period vacancy-posting cost

Notes: Impulse response functions (IRFs) represent the logarithmic deviation from steady state for each variable. IRFs are obtained by imposing a negative technology shock of $-\sigma = -0.0027$ to the log-linear model. The solid, blue IRFs are in the model with constant per-period vacancy-posting cost c. The dotted, red IRFs are in the model with per-period vacancy-posting cost $c \cdot a_t$ (model presented in the text). The time period displayed on the x-axis is 250 weeks.

The main differences with the moments reported in Table 3 concern the wage. The correlation between wage w and technology a is 0.668, in line with the empirical correlation of 0.627. The wage, however, responds less to technology shocks in the model than in the data. In the model, a 1-percent decrease in technology decreases wages by $\varepsilon_a^w = 0.668 \times 0.007/0.015 = 0.3\%$. In the data, a 1-percent decrease in technology decreases wages by 0.7%. To summarize, gradual wage adjustment increased the rigidity of wages but reduced the correlation of wages with technology.

As shown on the bottom panel of Table A3, increasing the coefficient γ does not solve the problem. With $\gamma = 0.8$ the volatility of labor market variables is much lower but the flexibility of wages measured by the elasticity $\varepsilon_a^w = 0.669 \times 0.008/0.015 = 0.35$ remains much too low (it is $\varepsilon_a^w = 0.7$ in US data). To conclude, introducing gradual wage adjustment improves the fit of the model along one dimension: wages are less correlated with technology. But it damages the fit along another dimension: wages are not flexible enough (the elasticity of wages with respect to technology is too low).

At the same time the moments of the labor market variables u, v, θ are remarkably similar to those reported in the text. This is because the present value of wages paid to each worker for the duration of a worker-firm match does not change much, even though the timing of wage payments is different. After a

negative shock the wage does not fall immediately: firm pays more at the beginning. Then the wage falls and remains lower than under the baseline specification for a while: the firm now pays less.

To better understand the dynamics of the model under this alternative wage-setting mechanism, I compute impulse response functions. Rationing unemployment is now given by

$$u_t^R = 1 - \left(\frac{\alpha \cdot a_{t-1}}{w_t} \right)^{1/(1-\alpha)}.$$

After log-linearization, it becomes

$$\check{u}_t^R = -\frac{1}{1-\alpha} \cdot \frac{1-\bar{u}^R}{\bar{u}^R} \cdot (\check{a}_{t-1} - \check{w}_t).$$

Figure A8 reports the IRFs, and compares them to those obtained in the baseline model. The IRFs confirm that (1) the dynamics of unemployment u, vacancy-unemployment ratio θ, and output y are not affected; (2) wages adjust gradually to the technology shock and do not jump down on impact, but the present value of wages paid to the worker (determined by the area between the x-axis and the IRF) does not change much; and (3) the impulse responses of rationing and frictional unemployment are larger on impact (because wages are more rigid on impact).

A4.4   Capacity-adjusted TFP series

Technology is not adjusted for variable factor utilization. Therefore, fluctuations in technology may be partly endogenous. To address this issue, I construct another series of model-generated unemployment using the quarterly, utilization-adjusted total factor productivity series (TFP) from Fernald [2009] as the model driving force. Actual and model-generated unemployment are shown on the bottom graph in Figure A9. The fit of the model remains good.

Figure A10 presents the decomposition using utilization-adjusted TFP series. The decomposition is similar to the one obtained with technology as a driving force. This result confirms the robustness of my finding that fluctuations in the composition of unemployment are large at business cycle frequency.

A4.5   HP filter with smoothing parameter of 1600

In the text I choose a smoothing parameter of 10^5 for the HP filter to make the numerical analysis comparable with the results of the literature [Barnichon, 2010; Costain and Reiter, 2008; Elsby and Michaels, 2008; Kennan, 2010; Mortensen and Nagypál, 2007; Rudanko, 2009; Shimer, 2005]. In this section I show that the results are not affected if I HP-filter quarterly US data and corresponding simulated series using a smoothing parameter of 1600. Table A5 reports simulated moments and Table A4 reports their empirical counterparts.

First, I estimate the new series for detrended log technology as an AR(1) process: $\ln(a_{t+1}) = \rho \cdot \ln(a_t) + z_{t+1}$ with $z_{t+1} \sim N(0, \sigma^2)$. I obtain $\rho = 0.982$ and $\sigma = 0.00264$ at weekly frequency. Then, I log-linearize the model around its steady state and perturb it with i.i.d. technology shocks $z_t \sim N(0, 0.00264)$.

Table A3: Simulated moments under technology shocks, with gradual wage adjustment

	u	v	θ	w	y	a
$\gamma = 0.7$						
Standard Deviation	0.093	0.114	0.202	0.007	0.019	0.015
Autocorrelation	0.904	0.774	0.866	0.975	0.841	0.820
Correlation	1	-0.899	-0.968	-0.716	-0.986	-0.979
	–	1	0.980	0.518	0.895	0.905
	–	–	1	0.621	0.960	0.962
	–	–	–	1	0.687	0.668
	–	–	–	–	1	0.998
	–	–	–	–	–	1
$\gamma = 0.8$						
Standard Deviation	0.054	0.066	0.117	0.008	0.017	0.015
Autocorrelation	0.900	0.768	0.861	0.976	0.825	0.810
Correlation	1	-0.895	-0.967	-0.718	-0.983	-0.978
	–	1	0.979	0.522	0.894	0.899
	–	–	1	0.625	0.959	0.959
	–	–	–	1	0.682	0.669
	–	–	–	–	1	0.999
	–	–	–	–	–	1

Notes: Results from simulating with stochastic technology the log-linearized model with gradual wage adjustment (A20). The top panel reports moments when wage flexibility is $\gamma = 0.7$. The bottom panel reports moments when wage flexibility is $\gamma = 0.8$. All variables are reported in log as deviations from an HP trend with smoothing parameter 10^5.
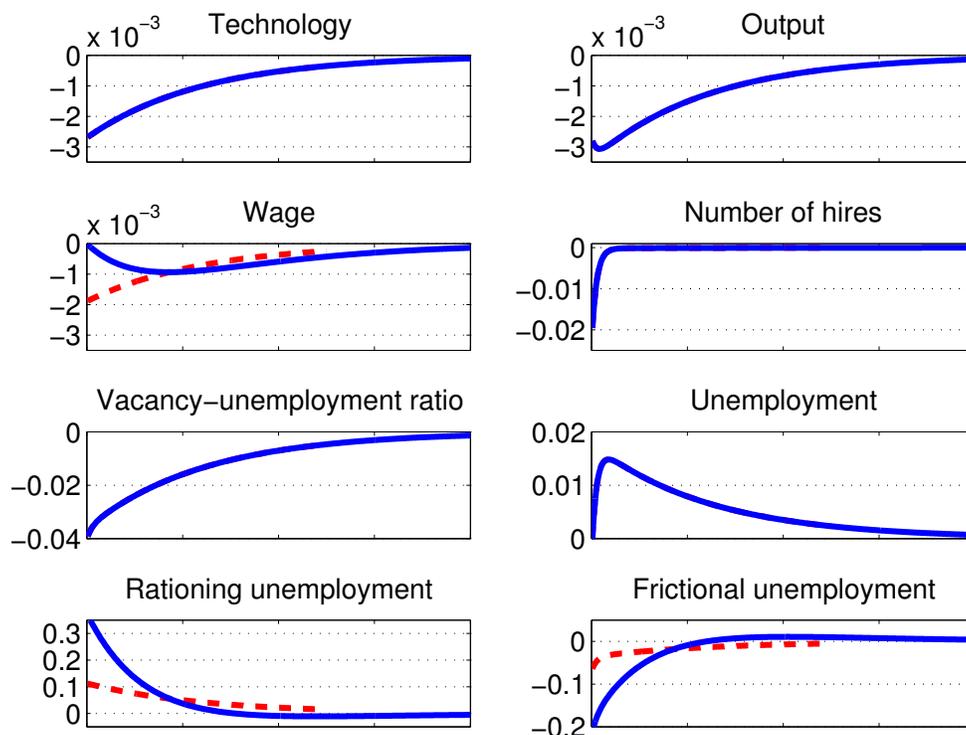
Figure A8: IRFs to a negative technology shock with gradual wage adjustment

Notes: Impulse response functions (IRFs) represent the logarithmic deviation from steady state for each variable. IRFs are obtained by imposing a negative technology shock of $-\sigma = -0.0027$ to the log-linear model. The time period displayed on the x-axis is 250 weeks. Solid blue line is under gradual wage adjustment (wage schedule (A20)). Dashed red line is under the baseline model from the text (wage schedule is $w_t = \omega \cdot a_t^\gamma$).

The main difference with the moments reported in Table 2 and Table 3 is that standard deviations are lower for actual and simulated variables. This was expected as a more important fraction of fluctuations is captured by the trend when the smoothing parameter is smaller (the trend is more volatile). I reach, however, similar conclusions about the fit of the model using the lower smoothing parameter of 1600:

- The model amplifies technology shocks roughly as much as observed in the data. In US data, a 1-percent decrease in technology increases unemployment by 5.0% and reduces vacancy by 6.7%. It reduces labor market tightness, measured by the vacancy-unemployment ratio, by 11.6%. In the model, a 1-percent decrease in technology increases unemployment by 4.2%, reduces vacancy by 4.8%, and reduces labor market tightness by 9.0%.

- Wages respond slightly more to technology shocks in the model than in the data. In the model, a 1-percent decrease in technology decreases wages by 0.7%. In the data, a 1-percent decrease in technology decreases wages by 0.5%. Increasing the wage rigidity (by reducing γ) would reduce the response of simulated wages and increase the response of simulated labor market variables, hence improving the fit of the model.
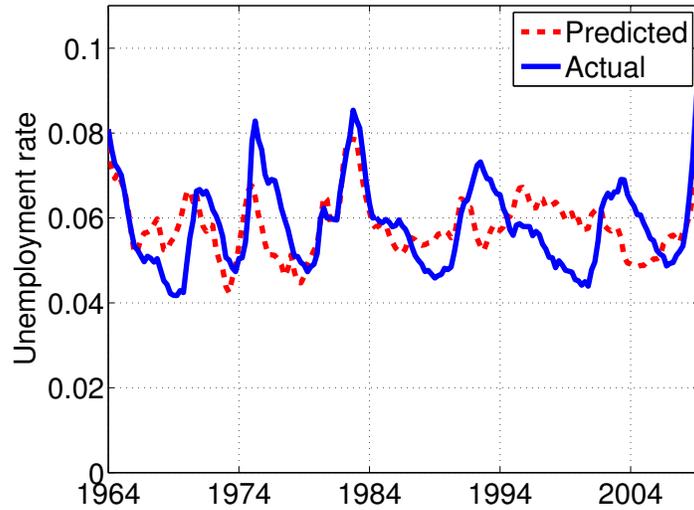
Figure A9: Actual unemployment, and unemployment generated from actual TFP, 1964–2009.

Notes: Actual unemployment is the quarterly average of seasonally-adjusted monthly series constructed by the BLS from the CPS. Predicted unemployment is generated when the model is stimulated by the quarterly, utilization-adjusted TFP series constructed by Fernald [2009]. Actual TFP, actual unemployment and predicted unemployment are detrended with a HP filter with smoothing parameter 10^5. The period is 1964:Q1–2009:Q2. I solve the nonlinear model with the Fair and Taylor [1983] shooting algorithm.
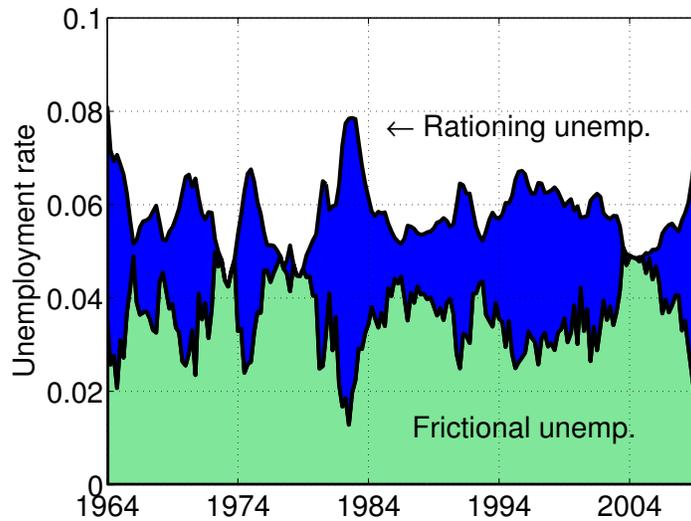
Figure A10: Decomposition of unemployment generated from actual TFP, 1964–2009.

Notes: The graph decomposes the unemployment series generated when the nonlinear model is stimulated by the detrended, quarterly, utilization-adjusted TFP series constructed by Fernald [2009]. The period is 1964:Q1–2009:Q2. I solve the nonlinear model with the Fair and Taylor [1983] shooting algorithm. Frictional and rationing unemployment are constructed from (18) and (19).

- Simulated and empirical slopes of the Beveridge curve are close. In the model the slope is -0.79, whereas it is -0.95 in the data.

- The main weakness of the model is that the correlations of labor market variables and wages with technology remain much too high compared with the data.

A5 Comparison of Different Numerical Solution Methods

Figure A11 compares the time series for unemployment and labor market tightness generated by the model with two different numerical solution methods: (i) a series of equilibria in static environments that abstract from aggregate shocks to technology and dynamics of unemployment; and (ii) the exact solution to the nonlinear model, which accounts fully for the dynamics of unemployment and rational expectations of stochastic process of technology and labor market variables.

The series of equilibria in a static environment is obtained by solving the system of three equations—definition of unemployment, Beveridge curve, and firm's optimality condition—to determine employment, unemployment, and labor market tightness for a series of technology levels. I plot the resulting unemployment and labor market tightness series.

The exact solution to the model is obtained by using the Fair and Taylor [1983] shooting algorithm. This algorithm solves dynamic rational expectation models period by period, by iterating in each period over the path of expected values for endogenous (employment and labor market tightness) and exogenous (technology) variables, until this path converges from an arbitrary path to a path of rational expectations, consistent with the predictions of the model.

While the time series obtained with these two numerical solution methods are quantitatively different, they are qualitatively similar. The main difference between these two labor market tightness series is that θ_t spikes and plummets more drastically with the steady-state solution method. This is because after a positive technology shock, firms do not take into account the fact that technology will eventually revert to a lower mean value, making recruiting less profitable. Therefore, firms are predicted to recruit too much in the steady-state solution method after a positive shock. For the same reason, firms are predicted to recruit too little after a negative shock, because they do not expect that technology will eventually revert to a higher mean value. This discrepancy in the predicted recruiting behavior of firms also affects predicted unemployment, but the difference between the two series is relatively small. To conclude, solving the model without accounting for aggregate shocks offers a good approximation to the exact solution.

A6 Calibration of Existing Search-and-Matching Models

I follow the same calibration strategy as in the text. All calibrated parameters are summarized in Table A6.

Table A4: Summary statistics, quarterly US data, 1964–2009. HP-parameter: 1600.

	u	v	θ	w	y	a
Standard Deviation	0.117	0.134	0.246	0.011	0.021	0.014
Autocorrelation	0.897	0.915	0.910	0.886	0.864	0.808
	1	-0.947	-0.984	-0.222	-0.859	-0.595
	–	1	0.989	0.319	0.907	0.701
Correlation	–	–	1	0.277	0.898	0.662
	–	–	–	1	0.439	0.555
	–	–	–	–	1	0.903
	–	–	–	–	–	1

Notes: All data are seasonally adjusted. The sample period is 1964:Q1–2009:Q2. Unemployment rate *u* is quarterly average of monthly series constructed by the BLS from the CPS. Vacancy rate *v* is quarterly average of monthly series constructed by merging data constructed by the BLS from the JOLTS and data from the Conference Board, as detailed in the text. Labor market tightness θ is the ratio of vacancy to unemployment. Real wage *w* is quarterly, average hourly earning in the nonfarm business sector, constructed by the BLS CES program, and deflated by the quarterly average of monthly CPI for all urban households, constructed by BLS. *y* is quarterly real output in the nonfarm business sector constructed by the BLS MSPC program. $\ln(a)$ is computed as the residual $\ln(y) - \alpha \cdot \ln(n)$ where *n* is quarterly employment in the nonfarm business sector constructed by the BLS MSPC program. All variables are reported in log as deviations from an HP trend with smoothing parameter 1600.

Table A5: Simulated moments with technology shocks. HP-parameter: 1600.

	u	v	θ	w	y	a
Standard Deviation	0.044	0.060	0.098	0.007	0.012	0.010
Autocorrelation	0.776	0.537	0.693	0.616	0.646	0.616
	1	-0.788	-0.926	-0.955	-0.967	-0.955
	–	1	0.962	0.804	0.788	0.804
Correlation	–	–	1	0.916	0.911	0.916
	–	–	–	1	0.998	1.000
	–	–	–	–	1	0.998
	–	–	–	–	–	1

Notes: Results from simulating the log-linearized model with stochastic technology. All variables are reported in log as deviations from an HP trend with smoothing parameter 1600.
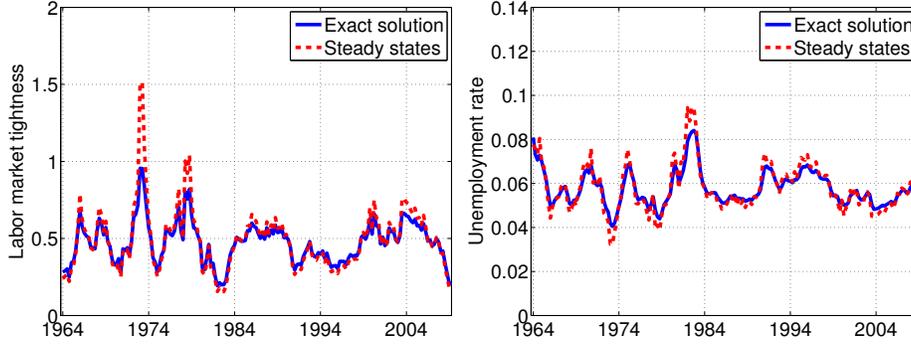
Figure A11: Labor market tightness and unemployment across solution methods.

Table A6: Parameter values for existing search-and-matching models (weekly frequency)

	Interpretation	Value	Source
Canonical model:			
c	Recruiting cost	0.32	$0.32\times$ steady-state wage
β	Worker's bargaining power	0.86	Matches unemployment $= 5.8\%$
Model with wage rigidity:			
c	Recruiting cost	0.32	$0.32\times$ steady-state wage
ω	Steady-state real wage	0.991	Matches unemployment $= 5.8\%$
Model with diminishing returns:			
c	Recruiting cost	0.22	$0.32\times$ steady-state wage
α	Returns to labor	0.21	Matches labor share$= 0.66$
β	Worker's bargaining power	0.86	Matches unemployment $= 5.8\%$

A6.1 Canonical model

In steady-state $c = 0.32 \times \overline{w}$, so the firm's optimality condition becomes

$$\frac{1 - \delta \cdot (1-s)}{q(\overline{\theta})} = \frac{1 - \overline{w}}{0.32 \cdot \overline{w}}.$$

I target $\overline{u} = 5.8\%$, or equivalently $\overline{\theta} = 0.45$. This pins down $\overline{w} = 0.990$, and $c = 0.32$. Then, in steady state, equilibrium condition (10) becomes

$$\frac{1 - \delta \cdot (1-s)}{q(\overline{\theta})} + \beta \cdot \delta \cdot (1-s) \cdot \overline{\theta} = (1-\beta) \cdot \frac{1}{c},$$

which pins down the bargaining power $\beta = 0.86$.

A6.2 Model with wage rigidity

In steady-state, $\overline{w} = \omega$ and $c = 0.32 \cdot \overline{w}$, so the firm's optimality condition becomes

$$\frac{1 - \delta \cdot (1 - s)}{q(\overline{\theta})} = \frac{1 - \omega}{0.32 \cdot \omega}.$$

I target $\overline{u} = 5.8\%$, or equivalently $\overline{\theta} = 0.45$. This pins down $\omega = 0.990$, and $c = 0.32$.

A6.3 Model with diminishing returns

Let $\kappa \equiv \alpha / [1 - \beta \cdot (1 - \alpha)]$. The steady-state wage equation, equilibrium condition (12), and definition of the labor share are

$$\overline{w} = \beta \cdot \left[\kappa \cdot \overline{n}^{\alpha - 1} + c \cdot (1 - s) \cdot \delta \cdot \overline{\theta} \right] \tag{A21}$$

$$(1 - \beta) \cdot \kappa \cdot \overline{n}^{\alpha - 1} = [1 - \delta \cdot (1 - s)] \frac{c}{q(\overline{\theta})} + c \cdot (1 - s) \cdot \delta \cdot \beta \cdot \overline{\theta} \tag{A22}$$

$$\overline{ls} = \overline{w} \cdot \overline{n}^{1 - \alpha}. \tag{A23}$$

Combining (A21), (A22), and (A23), and using $c = 0.32 \times \overline{w}$ yields:

$$\kappa = \left[[1 - \delta \cdot (1 - s)] \cdot \frac{0.32}{q(\overline{\theta})} + 1 \right] \overline{ls} \tag{A24}$$

$$\overline{ls} = \overline{w} \cdot \overline{n}^{1 - \alpha} \tag{A25}$$

$$\overline{w} = \beta \left[\kappa \cdot \overline{n}^{\alpha - 1} + c \cdot (1 - s) \cdot \delta \cdot \overline{\theta} \right]. \tag{A26}$$

Equation (A24) identifies $\kappa = 0.67$, as I target $\overline{ls} = 0.66$ and $\overline{\theta} = 0.45$. Equation (A25) determines $\overline{w} = 0.69$, as I target $\overline{n} = 0.95$. Finally, (A26) determines $\beta = 0.86$, and $\alpha = (\kappa - \kappa \cdot \beta)/(1 - \kappa \cdot \beta) = 0.21$.

References

Andolfatto, David, "Business Cycles and Labor-Market Search," *American Economic Review*, 1996, *86* (1), 112–132.

Barnichon, Regis, "The Lumpy Job Separation Rate: Reconciling Search Models with the Ins and Outs of Unemployment," 2010. http://sites.google.com/ site/ regisbarnichon/cv/Lumpyjs.pdf.

Blanchard, Olivier J. and Jordi Galí, "Labor Markets and Monetary Policy: A New-Keynesian Model with Unemployment," *American Economic Journal: Macroeconomics*, 2010, *2* (2), 1–30.

Costain, James S. and Michael Reiter, "Business Cycles, Unemployment Insurance, and the Calibration of Matching Models," *Journal of Economic Dynamics and Control*, 2008, *32* (4), 1120–1155.

Elsby, Michael W.L. and Ryan Michaels, "Marginal Jobs, Heterogeneous Firms, and Unemployment Flows," Working Paper 13777, National Bureau of Economic Research 2008.

Fair, Ray C. and John B. Taylor, "Solution and Maximum Likelihood Estimation of Dynamic Nonlinear Rational Expectations Models," *Econometrica*, 1983, *51* (4), 1169–1185.

Fernald, John G., "A Quarterly, Utilization-Adjusted Series on Total Factor Productivity," August 2009. http://www.frbsf.org/economics/economists/jfernald/Quarterly-TFP.pdf.

Gertler, Mark and Antonella Trigari, "Unemployment Fluctuations with Staggered Nash Wage Bargaining," *Journal of Political Economy*, 2009, *117* (1), 38–86.

_ , Luca Sala, and Antonella Trigari, "An Estimated Monetary DSGE Model with Unemployment and Staggered Nominal Wage Bargaining," *Journal of Money, Credit and Banking*, 2008, *40* (8), 1713–1764.

Haefke, Christian, Marcus Sonntag, and Thijs Van Rens, "Wage Rigidity and Job Creation ," Discussion Paper 3714, Institute for the Study of Labor (IZA) 2008.

Hall, Robert E., "Employment Efficiency and Sticky Wages: Evidence from Flows in the Labor Market," *Review of Economics and Statistics*, 2005, *87* (3), 397–407.

_ and Paul R. Milgrom, "The Limited Influence of Unemployment on the Wage Bargain," *American Economic Review*, 2008, *98* (4), 1653–74.

Kennan, John, "Private Information, Wage Bargaining and Employment Fluctuations," *Review of Economic Studies*, 2010, *77* (2), 633–664.

Mortensen, Dale T. and Éva Nagypál, "More on Unemployment and Vacancy Fluctuations," *Review of Economic Dynamics*, 2007, *10* (3), 327–347.

Petrongolo, Barbara and Christopher A. Pissarides, "Looking into the Black Box: A Survey of the Matching Function," *Journal of Economic Literature*, 2001, *39* (2), 390–431.

Pissarides, Christopher, "Unemployment and Vacancies in Britain," *Economic Policy*, 1986, *1* (3), 500–559.

Pissarides, Christopher A., *Equilibrium Unemployment Theory*, 2nd ed., Cambridge, MA: MIT Press, 2000.

_ , "The Unemployment Volatility Puzzle: Is Wage Stickiness the Answer?," *Econometrica*, 2009, *77* (5), 1339–1369.

Rotemberg, Julio J., "Cyclical Wages in a Search-and-Bargaining Model with Large Firms," in "NBER International Seminar on Macroeconomics 2006" NBER Chapters, National Bureau of Economic Research, Inc., 2008.

Rudanko, Leena, "Labor Market Dynamics Under Long-Term Wage Contracting," *Journal of Monetary Economics*, 2009, *56* (2), 170–183.

Sala, Luca, Ulf Soderstrom, and Antonella Trigari, "Monetary Policy under Uncertainty in an Estimated Model with Labor Market Frictions," *Journal of Monetary Economics*, 2008, *55* (5), 983–1006.

Shimer, Robert, "The Cyclical Behavior of Equilibrium Unemployment and Vacancies," *American Economic Review*, 2005, *95* (1), 25–49.

_ , "Reassessing the Ins and Outs of Unemployment," October 2007.

Thomas, Carlos, "Search and Matching Frictions and Optimal Monetary Policy," *Journal of Monetary Economics*, 2008, *55* (5), 936–956. Carnegie-Rochester Conference Series on Public Policy: Labor Markets, Macroeconomic Fluctuations, and Monetary Policy November 9-10, 2007.