MODELING REPUTATION-BASED BEHAVIORAL BIASES IN SCHOOL CHOICE

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FACTS OF LIFE

College applications are complicated — take a breath
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TWO SOURCES OF UTILITY

consumption utility **+** **subjective value**

- classical term
- **benefit** from the **single** school the student ultimately attends

- models emotional effect
- **benefit or loss** from **every** school to which the student applies

[Kőszegi & Rabin 2006, 2007, 2009]
[Dreyfuss, Heffetz, Rabin 2022]
[Meisner & von Wangenheim 2023]

LINEAR THRESHOLD MODEL

single student with score $s \sim U[0,1]$ (realization observed by schools, not student)
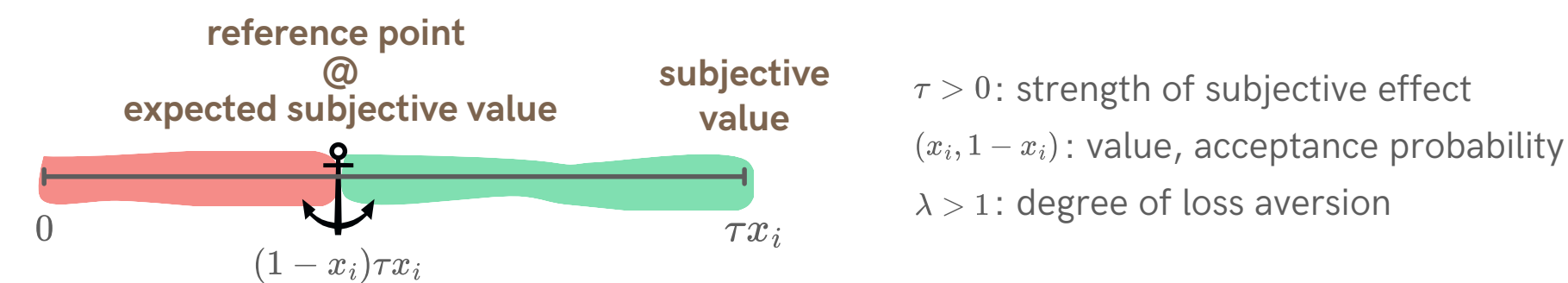
continuum of schools $S = [0,1]$

- for all $x \in [0,1]$, there exists a school with both **value** and **acceptance threshold** x
- student is accepted to school x iff $s \geq x$ (w.p. $1-x$)

0   accepted   rejected   1

[Ali & Shorrer 2023]

BIASED UTIILTY FUNCTION

reference point @ expected subjective value

subjective value

$(1 - x_i)\tau x_i$ τx_i

$\tau > 0$: strength of subjective effect
$(x_i, 1 - x_i)$: value, acceptance probability
$\lambda > 1$: degree of loss aversion

- if accepted, **subjective benefit** $\tau x_i - (1 - x_i)\tau x_i = \tau x_i^2$
- if rejected, **subjective loss** $-\lambda(1 - x_i)\tau x_i$
- ⇒ **expected subjective utility**

$(1 - x_i)\tau x_i^2 + x_i(-\lambda(1 - x_i)\tau x_i)$
$= \tau(1 - x_i)x_i^2(1 - \lambda)$
$= -\gamma(1 - x_i)x_i^2$

bias parameter
$$\gamma = (\lambda - 1)\tau > 0$$

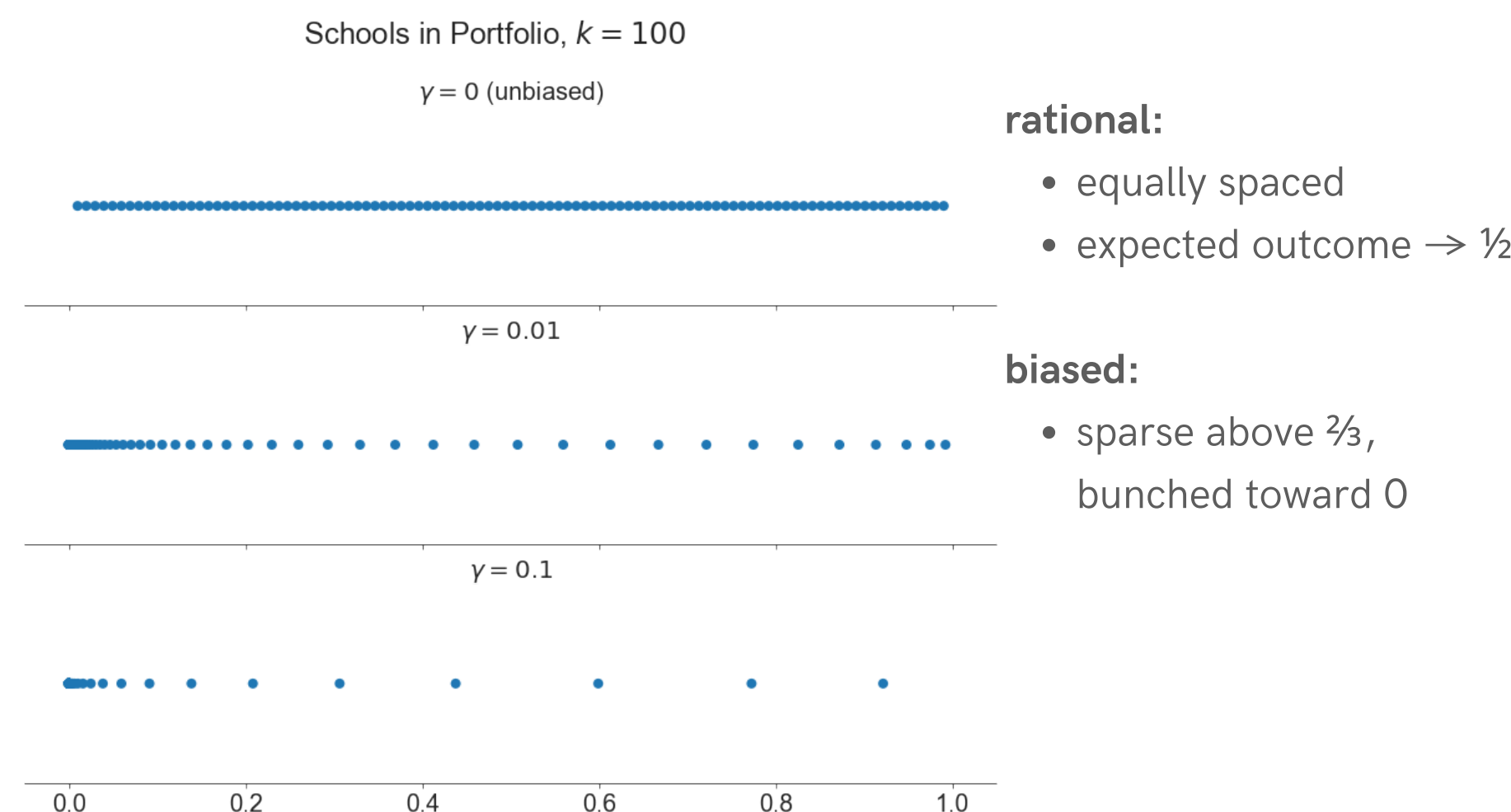**perceived expected utility of a portfolio** $x_1 > x_2 > \cdots > x_k$:

0   x_k $\bullet\bullet\bullet$ x_2 x_1 1

$$U_\gamma(x_1, \ldots, x_k) = \sum_{i=1}^{k} \left[-\gamma(1 - x_i)x_i^2 + x_i(x_{i-1} - x_i) \right]$$

subjective   consumption

FOC: $x_i - x_{i+1} = x_{i-1} - x_i - \gamma(2x_i - 3x_i^2)$

$\underbrace{}_{\Delta_{i+1}}$ $(x_i > \frac{2}{3})$ $>$ $<$ $(x_i < \frac{2}{3})$ $\underbrace{}_{\Delta_i}$

SOLUTION STRUCTURE

Schools in Portfolio, $k = 100$

$\gamma = 0$ (unbiased)

$\gamma = 0.01$

$\gamma = 0.1$

rational:
- equally spaced
- expected outcome → ½
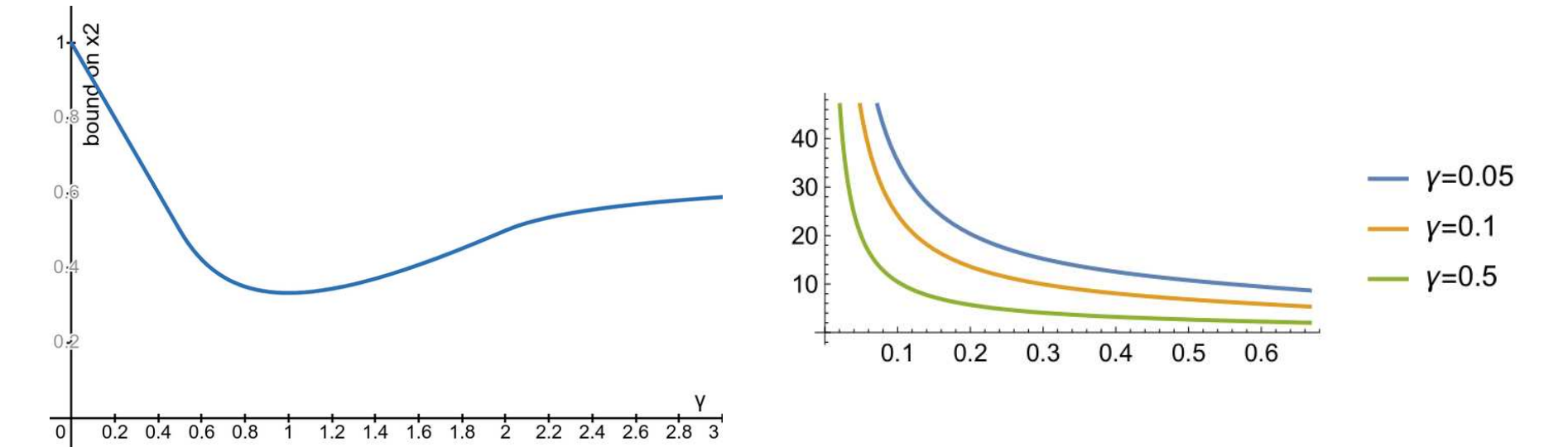
biased:
- sparse above ⅔, bunched toward 0

THEOREM 1.
As $k \to \infty$, a biased student's expected outcome converges to $\frac{1}{2} - \Theta(\sqrt{\gamma})$.
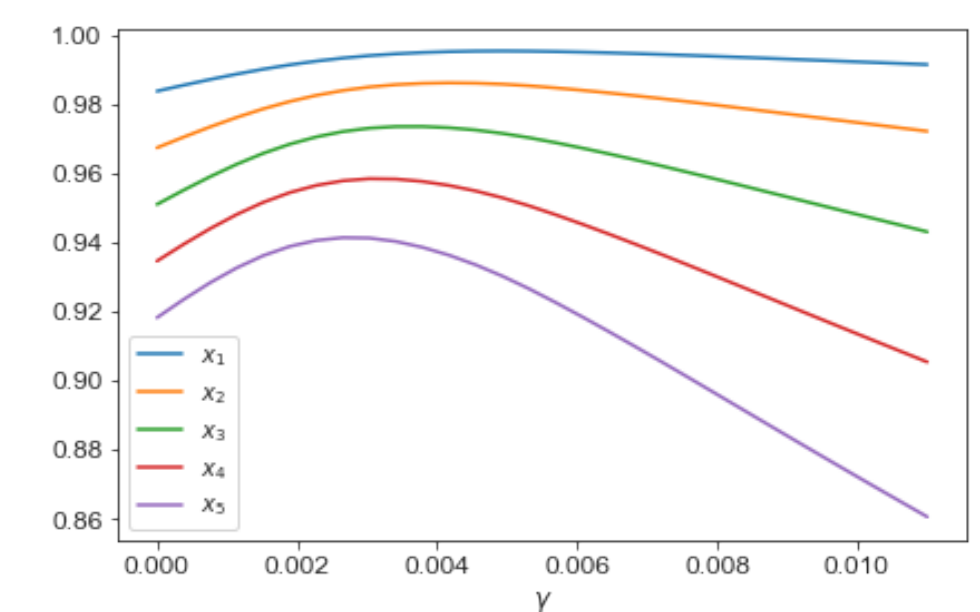
MISTAKE: UNDERAPPLYING

THEOREM 2.
For any k, a biased student's second highest school x_2 is strictly below 1.

THEOREM 3.
For any k and constant c, a biased student applies to at most $O\left(\frac{1}{\sqrt{\gamma}}\right)$ schools in $[c, 1]$.

γ=0.05
γ=0.1
γ=0.5

A SECOND SURPRISE

x_1
x_2
x_3
x_4
x_5

THEOREM 4.
There exist portfolio sizes k such that for any sufficiently small γ, $x_1 > \frac{k}{k+1}$.

CONCLUSIONS

small amounts of **bias** ⟶ { **large complex counterintuitive** } shifts in **behavior**

OPEN DIRECTIONS

within model: general distributions, relaxing correlation, nonlinear utilities
further extensions: multi-student games, exploration and information acquisition, timing and signaling

REFERENCES
- Jon Kleinberg, Sigal Oren, Emily Ryu, and Éva Tardos. Modeling reputation-based behavioral biases in school choice, 2024. https://arxiv.org/pdf/2403.04616